PROCEEDINGS

OF THE

ROYAL SOCIETY OF EDINBURGH.

VOL. VIII. 1874–75. No. 90.

Ninety-Second Session.

Monday, 23d November 1874.

Sir **WILLIAM THOMSON**, President, in the Chair.

The following Council were elected :—

President.
Sir **WILLIAM THOMSON**, Knt., LL.D.

Honorary Vice-Presidents.
His Grace the **DUKE of ARGYLL**.
Sir **ROBERT CHRISTISON**, Bart., M.D.

Vice-Presidents.
Sir W. Stirling-Maxwell, Bart.
David Milne Home, LL.D.
Professor Kelland.
Rev. W. Lindsay Alexander, D.D.
David Stevenson, Esq., C.E.
The Hon. Lord Neaves.

General Secretary—Dr John Hutton Balfour.

Secretaries to Ordinary Meetings.
Professor Tait.
Professor Turner.

Treasurer—David Smith, Esq.

Curator of Library and Museum—Dr Maclagan.

Councillors.
Professor Lister.
George Robertson, Esq., C.E.
The Right Rev. Bishop Cotterill.
Professor A. Crum Brown.
Dr Arthur Mitchell.
George Forbes, Esq.
Principal Sir Alex. Grant, Bart.
Professor Geikie.
Dr Andrew Fleming, H.M.I.S.
Dr Charles Morehead.
Alexander Buchan, A.M.
Robert Wyld, Esq.

BRITISH MUSEUM

Monday, 7th December 1874.

Sir WILLIAM THOMSON, President, in the Chair.

The Keith Prize for the Biennial Period (1871–73) having been awarded by the Council to Professor Tait for his Paper, entitled "First Approximation to a Thermo-Electric Diagram," which has been published in the Transactions, the Medal was delivered to him by the President at the commencement of the meeting.

The President said—I have now the pleasing duty of awarding the Keith Medal, for 1871–73, to Professor Tait, for a paper entitled "First Approximation to a Thermo-Electric Diagram." This paper is published in the number of the Transactions which I hold in my hand, being Part I. of Volume XXVII., for Session 1872–73. The Society, considering the remarkable interest which attaches to the subject of this paper, and the very important discovery which it contains, will, I am sure, pardon me if I take up a little of their time by referring to some of the antecedents of Professor Tait's investigation. In the first place, there was the great discovery by Seebeck, about the year 1821, of thermo-electric currents. According to this discovery, if a circuit be formed of two different metals, and if the two junctions be kept at different temperatures, an electric current will be found to go round the circuit:—that is Seebeck's great discovery. Quickly following upon it was the very remarkable discovery made by Cumming, the late professor of chemistry in the University of Cambridge, that there is in certain pairs of metals an inversion of the thermo-electric current of a very remarkable kind. A few words are necessary to make it intelligible. Take a circuit of, let us say, bismuth and antimony. Let one of the junctions be kept at an ordinary temperature. Let the other junction be gradually raised in temperature. With a proper instrument for measuring the strength of the current flowing through the circuit, it will be found that the strength of the current gradually increases as the hot junction is made hotter and hotter. That was part of Seebeck's discovery.

Cumming experimented on many other pairs of metals, and, amongst them, copper and iron. In experimenting upon copper and iron, he found that as the hot junction is made hotter and hotter, the current increases, but only up to a certain limit. When the hot junction is made still hotter than a certain critical temperature, the current begins to diminish in strength, till it becomes zero. When the hot junction is made hotter still, the current becomes reversed. This was a very great discovery indeed in thermo-electricity. I could be tempted to go into some more details, but I should perhaps tax the patience of the Society were I to do so. But I cannot refrain from mentioning the thermo-dynamic relations of this discovery and the collateral discovery by Peltier, that there is a thermal effect produced at a junction of two different metals when by any means an electric current is caused to cross from one metal to another. I call that a collateral discovery,—collateral with Seebeck's, because it might have been made quite independently of Seebeck's. That might be one discovery and Seebeck's another, thoroughly disconnected, had we not light from theory to put them into relation with one another. The first ray of light thrown on the subject is to be found in an almost casual remark made by Joule in the course of some not casual—but thoroughly worked-out—observations upon the relations between the generation of heat on the one hand and the development of work from heat on the other. He remarked that the absorption of heat or the production of cold, manifested under certain circumstances by electric currents crossing a junction of dissimilar metals in Peltier's discovery, was to be looked to as a source of the power to be developed from a current of electricity produced by a thermo-electric action. The thorough working out of this remark of Joule's required the application of Carnot's theory. The relation between heat absorbed in one junction of the thermo-electric circuit, heat evolved in the other, and work done by the current in any instrument driven by the current put into the circuit—let us say, a mechanical engine raising weights, driven by the current—the relation, I say between heat absorbed in the one junction, heat developed in the other, and mechanical work performed, was fully worked out by the application of Carnot's theory. The introduction of Cumming's inversion led to a very novel idea regarding the properties of matter, and, I may say, a novel discovery of action in connection with heat and electricity. To this action the name of the electric convection of heat has been given. When this is taken into account in connection with the discoveries of Peltier, Cumming, and Seebeck, it appears that the heat supply by which an engine driven by the thermo-electric current gets its energy may be not in a junction between two dissimilar metals, but by the peculiar absorption of heat, or generation of heat, which the discovery I have last mentioned points at when a current passes from hot to cold or cold to hot in the same metal. I am afraid the subject is somewhat involved; but I can only say that if the Fellows of the Royal Society diligently read their own Transactions, it would not be necessary for me to speak about it, because it was very minutely unfolded in a paper published a good many years ago. I chanced to be the author of that paper myself, and I speak just now, I must confess, with a considerable pride of the position it bears in relation to Professor Tait's discovery. This discovery is, that there are in many cases no doubt, but notably in cases in which iron is one of the metals, not merely one neutral point, as discovered by Cumming, but several neutral points. Tait finds that when a circuit, composed of iron and (let us suppose) platinum, has the temperature of one junction gradually elevated, while that of the other is maintained constant, the current first increases, then diminishes, then increases, then diminishes,—or, it may be, first increases, then diminishes to nothing, and goes in the contrary direction, attains a maximum in that contrary direction, then diminishes from that maximum to zero, and increases in the first direction to a maximum, and so on with every alternation. "Multiple neutral points" is the shortest name I can give in words to the great discovery in Professor Tait's paper. But I would do Professor Tait great injustice, and give you a very imperfect idea of the substantial character of the investigation for which the award has been made, were I to lead you to suppose that it was merely a discovery which some of you might for a moment imagine could be hit off by chance, and by an experimenter who had not his eyes absolutely closed. This discovery was made in the course of an elaborate investigation, of which results have been published from year to year, and generally within the two years for which the award has been given, in the Proceedings of this Royal Society. From year to year Professor Tait had accurate measurements of thermo-electric currents executed in his laboratory. The results he tabulated and represented graphically; and even before making this discovery, he had made some exceedingly important contributions towards the laws of thermo-electric force in various combinations of metals. It was in pursuing this elaborate investigation that he came upon the very astounding discovery—I call it really astounding—that there are several consecutive neutral points between one and another of certain pairs of metals—iron, platinum, iridio-platinum. It is not my object to give you imperfectly a small part of the information contained in this paper of Tait's. I desire rather to call your attention to a thing of much value which you have in your Transactions. But I may be allowed the pleasure of referring to the relation which my own previous investigation bears to this discovery of Tait's. I did succeed in doing away with the Peltier effect at one junction; I showed a thermo-electric current without the Peltier effect at one junction but with Peltier effect at the other junction, and the counter effect, absorption or evolution as it might be, made by my own thermo-electric convection in the homogeneous parts of the circuit. Now, Tait gives us a circuit in which there is no Peltier effect at all, and in which the whole origin of the power is to be found in absorption on the one hand and evolution on the other through electric convection of heat. I am afraid I have taxed your patience too long, because these matters have been reported in the Proceedings of the Society, and we have now a combined statement of the results in this paper. Before concluding, I wish to call your attention to the title of the paper, "A Thermo-electric Diagram," and the admirable manner in which Professor Tait has represented those results graphically to the eye, and the good use he has made of that graphical representation in aiding him to work out rigorously the laws of the phenomenon. I am sure the Society will feel very great satisfaction in the award which has been made on the present occasion. (To Professor Tait) I have now much pleasure in presenting you with the Keith Medal. If it were necessary that there should be an inducement to you to continue your labours,—I cannot say with increased zeal,—but with zeal equal to that with which you have carried them on for the last few years, let me hope that the presentation of this medal may form a small part at least in such a motive.

The President opened the Session with an Address on Stability of Steady Motion.

The following statement respecting the Members of the Society was read by the President:—

I. Honorary Fellows—		
Royal Personage,	1	
British Subjects,	20	
Foreign,	30	
Total Honorary Fellows,		51
Foreign Honorary Fellows Deceased since Session 1872–73: — Agassiz, Elie de Beaumont, Guizot, Hansteen, Quetelet, and De la Rive,	6	
II. Non-resident Member under the Old Laws,		1
III. Ordinary Fellows—		
Ordinary Fellows at November 1873,	346	
New Fellows, 1873–74.—Dr John Anderson; W. F. Barrett, Esq.; Dr Joseph Bell; Thomas Muir, M.A.; Dr Benjamin Carrington; Dr John Chiene; William Durham, Esq.; William Ferguson, Esq.; Dr Alexander Hunter; A. Forbes Irvine, Esq.; Francis Jones, Esq.; Dr E. A. Letts; James Napier, Esq.; T. B. Sprague, M.A. Cantab.; Dr R. H. Traquair; Dr J. Batty Tuke,	16	
		362
Carry forward,		362

Brought forward,		362
Deduct Deceased—Professor Thomas Anderson; George Berry, Esq.; Adam Black, Esq.; Sheriff Cleghorn; The Right Hon. Lord Colonsay; Francis Deas, Esq.; Henry Dircks, Esq.; William Euing, Esq.; Dr Robert Grant; Professor John Hunter; Professor Cosmo Innes; Charles Lawson, sen., Esq.; and Henry Stephens, Esq., ...	13	
Resigned—Dr W. M. Buchanan, Dr Alex. Wood, .	2	
Cancelled—Alfred R. Catton, Rev. Dr Hodson, .	2	
		17
Total number of Ordinary Fellows at Nov. 1873, .		345
Add Honorary and Non-Resident Fellows, . .		52
Total to 1874,		397

Monday, 21st December 1874.

PROFESSOR KELLAND, Vice-President, in the Chair.

The following Communications were read:—

1. Remarks on the Great Logarithmic and Trigonometrical Tables computed in the Bureau du Cadastre under the direction of M. Prony. By Edward Sang.

The volume marked "Logarithms O," now placed on the Society's table contains, to 28 places, the Logarithms of all numbers up to Ten Thousand. In preparing it, care was taken that each prime should be placed in connection with two distinct sets of other prime numbers, in order that all likelihood of mistake should be avoided. This amounts to the computation of each logarithm by two independent processes. Opposite each prime, reference is made to the page of the record of calculations; and the accompanying volumes marked "Construction, I., II.," contain minutes of each articulate step, so that the genesis of any logarithm may readily be traced and examined.

The logarithms of the composite numbers have been found by addition; the greater part of these have been twice computed, but the whole await a revision in the manner about to be described.

Though very unlikely, it is yet possible that the same error may happen in each of the two calculations; thus in computing the logarithm of 8447, two distinct formulæ were used, namely—

$$2 \cdot 3769 \cdot 10^8 + 1 = 3 \cdot 37 \cdot 251 \cdot 3203 \cdot 8447$$
$$643 \cdot 10^7 - 1 = 3 \cdot 89 \cdot 2851 \cdot 8447$$

and the agreement of the two results would show conclusively both to be right, were it not for the possibility of a coincidence of error in some of the other logarithms, or of an error in the logarithm of 3 which is common to both.

In order to remove this exceedingly slight doubt, I prepared an index on movable cards, to show the page on which each formulæ is found; and another index, laid on the table, to show the formulæ into which each prime enters; this list, however, being limited to the first five occurrences. The above prime is there found in two more combinations:—

$$2017 \cdot 10^6 + 1 = 73 \cdot 3271 \cdot 8447$$
$$8447 \cdot 10^4 + 1 = 3 \cdot 7 \cdot 11 \cdot 37 \cdot 9883,$$

which also consist with previous determinations. Such coincidence removes all doubt as to the accuracy of the logarithm of 8447, excepting, indeed, the last-place uncertainty incident to all computations of incommensurable quantities.

To put the last touch to the trustworthiness of this table, I had begun to supply, from the larger primes downwards, a third formula to each prime having only two marked against it; that is, to place each prime in at least three distinct combinations, with the intention, when this revision shall have been completed, to cause the whole of the logarithms of the primes to be re-extracted from the minutes, and the summations to be redone by another hand; when my attention was drawn to a note in the scientific periodical "Nature," in reference to the proposed publication of the Million

Table for which this Vol. O, though then incomplete, had served as the foundation. This note I recite in full:—

Note from "Nature," 8th October 1874.

"The President and Council of the Royal Society of Edinburgh, impressed with the conviction that the progress of the sciences demands, and has long demanded, fuller and more exact tables of logarithms than any which at present exist, have memorialised Sir Stafford Northcote with the view of inducing the Government to print a nine-figure table of logarithms from unity to a million, part of which has already been calculated by Mr Sang, who has carried a fifteen-figure table up to 300,000. The subject of undertaking the publication of logarithm tables—so long as the number of figures does not exceed ten, the limit of utility—is one well worthy the attention of the Government; but in the present case there are several reasons why, if the application is refused, the loss to science will not be so great as some might think. In the first place, a table of 1800 large pages, whether in one, two, or three volumes, will be so unwieldy that, notwithstanding the ease of the interpolations, it would probably be very seldom used by computers; and secondly, because all who require more than seven figures will, no doubt, prefer to use ten, and consult the existing works. In fact, nearly all computors would, we believe, employ Vlacq or Vega in preference to the proposed table. Mr Sang, in the pamphlet which accompanies the memorial, makes a remarkable error when he intimates that the great French tables have not been used to verify any seven-figure table, so that, 'up to the present moment we have no verification of Vlacq's great work.' In point of fact, the whole of Vlacq was read with the copy of the French tables, at the Paris Observatory, by M. Lefort, and the results are published in Vol. IV. of the 'Annales de l'Observatoire de Paris.' Almost all the errors found by Mr Sang, by means of this table are among those there given by Lefort, and any one who choses can, without much expenditure of trouble, render his copy of Vlacq all but free from error—much more accurate than any new table could possibly be."

In my paper on last-place errors in Vlacq, read here on the 20th April, I say, speaking of the *Tables du Cadastre*, "I have not learned that these computations have been used for the verification of those already printed, ... " and I have now to thank, very cordially, the author of the note for having mentioned the labours of M. Lefort, and so put me in the way of obtaining very startling and very important information in regard to Prony's Tables.

Concerning the errors in Vlacq, the author of the note says, "Almost all the errors, found by Mr Sang by means of this table are among those there given by Lefort," and deduces from this fact, in accordance with the well-established axiom that the part is *greater* than the whole—the futility of this or of any other new table. Not having yet mastered the first principles of this system of logic, I shall not venture to discuss any of the opinions of its inventor, and shall only look at the wisdom of his conclusion.

The studious investigator living in Iceland, in Terra del Fuego, or even here in Edinburgh, where logarithms had their birth, needing an extensive table of logarithms, must apply to his bookseller for Vlacq; the book has been "out of print" for two hundred years; if found at all, its price is antiquaries' price. Having succeeded, his next business is to procure a copy of the volume of the "Annales de l'Observatoire." I myself have tried the libraries here in vain, so that "without much expenditure of trouble," I have not made my copy of Vlacq "*all but* free from error,—much more accurate than any new table could *possibly* be." Even after I shall have effected the correction by help of Lefort's list, there will remain a great uncertainty, arising from the fact that two copies of Vlacq may not be in accordance with each other. To understand this, we may turn to the Errata printed on page 64 of Taylor's Tables. There we find, among others, the following remarks:—

"In about 100 copies; in about 120 copies; doubtful whether a few copies are erroneous or not; in about half the impression; only in one copy; and so on."

The movable types had been drawn out by the inking dabber, and erroneously replaced by the pressman. But in this case there is another uncertainty. Complaints were made of pirated editions, *fac simile* of the original Vlacq.

The search for the list of errors mentioned in the above note, led me to find two papers by M. F. Lefort. The first of these, contained in the "Comptes Rendus," tome xliv. page 1097, was read to the Academy on the 25th May 1857, and entitled "Note sur les erreurs que contient une des Tables de Logarithmes de Callet, par M. F. Lefort," shows its author to possess every claim to our confidence and respect as a computer. The indication of wholesale errors in Callet's twenty-place table (copied also into Hutton's) is invaluable, as preventing farther mistake, and as showing the absolute need for a revision of our most trusted tables.

The second paper is in vol. xlvi., and was occasioned by remarks on the presentation to the Library of the Institute of a manuscript copy of the great tables which had been in the possession of Prony himself. It was presented by Prony's heirs, at the meeting of the Academy, on Monday the 17th May 1858.

In the course of remarks after the presentation, M. Elie de Beaumont expressed his opinion that the best means of preserving the work would be to print it; and M. Leverrier, drawing attention to M. Lefort's labours, spoke as if a doubt exist even as to the whereabouts of the veritable original calculations.

At the next meeting, that of the 24th May 1858, the note above mentioned was read, entitled, "Note sur les deux exemplaires manuscrits des Grandes Tables logarithmiques et trigonométriques, calculées au Bureau du Cadastre, sous la direction de Prony; par M. F. Lefort."

I regret that the length of this most remarkable and most valuable memoir precludes its reproduction here. As, however, the "Comptes Rendus" are very widely distributed, a verification of my remarks upon it is within the reach of many. The note refers almost exclusively to the logarithmic part of the tables.

It tells us that the great work was accomplished by a staff of computors, divided into three sections; the first section consisting of four or five geometers, whose business was to do the purely analytical part, and to calculate certain fundamental numbers; the second section was formed of seven or eight calculators acquainted with analysis: they made calculations directly from the formulæ arranged by the first section. The third section contained seventy to eighty persons having a very slight acquaintance with mathematics, whose business was to perform the additions and subtractions prescribed by the second section.

We have here a little army, with its generals and lieutenants, M. Prony himself being commander-in-chief. Lefort does not acquaint us with the duties undertaken by the director, neither does he indicate the nature of the logarithmic formulæ which needed the concurrence of four or five geometers for their establishment. The actual business described by him begins with the doings of the second section, composed of "sept on huit calculateurs, possédant l'analyse et ayant une grande pratique de la traduction des formules en nombres."

These calculated 1°, the ten thousand first logarithms to nineteen decimals.

In regard to this part of the work, M. Lefort supplies us, somewhat indirectly however, with some very distinct information. He says (page 996) that Prony had borrowed a copy of Briggs' *Arithmetica Logarithmica*, which he returned enriched with an "*errata*," preceded by a note to the following effect;—

"This 'errata' is composed 1° of that which is at the top of the Latin introduction (of Briggs); 2° of the faults found by the citizens Letellier and Guyétant, calculators in the Bureau du Cadastre, on collating Briggs' table with the great tables of the Cadastre. These latter faults are marked by (a particular sign) the sign *."

Lefort goes on to say that this collation brought out thirty-two new faults, of which, however, he finds that *four* belong to the Cadastre table, leaving twenty-eight only to Briggs. Farther on he tells us that all the corrections for numbers above 10,000 refer to figures within eleven decimal places, leaving the twelfth, thirteenth, and fourteenth places of Briggs unchecked. From this I understand, although it be not explicitly so stated, that within the ten thousand, even these figures in Briggs had been examined.

Now, here I shall take the liberty of making an interjection of my own.

In March, while comparing my fifteen-place table with Vlacq, I thought it desirable to examine also the last places of Briggs; so the final groups of four figures in the *Arithmetiea Logarithmica* were read with the corresponding figures in the volume now on the table, up to 3000. The readers found the labour of recording the discrepancies so great that they had to content themselves with a pencil mark when the difference was only unit, and with a double mark when the divergence was more. Moreover, since perusing M. Lefort's note, I have caused the same readers to examine the tenth thousand, and I exhibit the book itself with their pencil marks. On counting these, it is found that considerably over forty per cent. of Briggs' final figures are in error.

When we consider that Briggs used only fifteen decimals in his "Tabula inventioni Logarithmorum inserviens," we can hardly expect a smaller proportion of errors than this among his final figures; and the examination exhibits in a very strong light the scrupulous care bestowed by him upon the work. But M. Prony's coadjutors of the second section carried their operations to nineteen decimals—that is, to one hundred thousand times the exactitude aimed at by Briggs, and not one of his errors should have escaped detection. It becomes, then, quite a mystery how MM. Letellier et Guyétant should have allowed upwards of four thousand errors to escape their notice.

To resume. Having accomplished this first part of their labour, the members of the second section computed 2° "the logarithms from 10,000 to 200,000 by intervals of 200 to 14 decimals, and with four, five. and even six orders of differences. The number of decimals was successively augmented by two for each order; so that, for example, the sixth difference was written with 26 decimals;" and this seems to have concluded their labours so far as the logarithmic table is concerned. The results were handed over to the third section.

Again, leaving for a while the course of M. Lefort's details concerning the preparation of the ruled sheets, I shall begin in earnest the computation of the final table. The logarithm of 10,000, and the differences of the successive orders, are inscribed on the upper horizontal line of a sheet, as in the annexed example. Our business is to add the first difference to the accompanying logarithm; to take the second difference from the first; the third from the second, and so on; and this has to be repeated 200 times, until we reach the number 10,200, whose logarithm, with its differences, ought to agree with what has been prepared for the second sheet. There are then to be performed two hundred additions and one thousand subtractions of large numbers before the calculator of the third section arrive at a check on, or can know anything of, the accuracy of his work. Also, an error in the determination of the first difference of the sixth order is augmented 82,472,326,300 times in the final logarithm. Perhaps it is on this account that the differences are carried two places further at each step.

I do not understand in what way the two additional figures belonging to one order of difference are disposed of when subtracted from the difference of the next lower order, and M. Lefort gives us no information on this most perplexing subject. Putting the embarrassment therewith connected out of view, the performance of these twelve hundred operations, subject at every step to the prodigious accumulation of early error, is a task which, I venture to affirm, was never successfully accomplished by any computer.

This, however, is but a small part of the difficulty. In order to make the matter clear, I have placed in the accompanying scheme the logarithims of the first ten numbers, with their differences arranged in the manner described, as they ought to be found on the first sheet, each of them true to the last figure.

N.	Log.	1st Diff.	2d Diff.	3d Diff.	4th.	5th.	6th.
10000	·00000 00000 0000	4 34272 76862 7	43 42076 382	868 19823	26036 83	10 4100	520 2
01	04 34272 7686	4 34229 34786 3	43 41208 184	867 93786	26026 42	10 4048	519 9
02	08 68502 1165	4 34185 93578 1	43 40340 246	867 67760	26016 02	10 3996	519 6
03	13 02688 0523	4 34142 53237 9	43 39472 568	867 41744	26005 62	10 3945	519 2
04	17 36830 5846	4 34099 13765 3	43 38605 151	867 15738	25995 23	10 3893	519 0
10005	21 70929 7223	4 34055 75160 1	43 37737 994	866 89743	25984 84	10 3841	
06	26 04985 4739	4 34012 37422 1	43 36871 096	866 63758	25974 45		
07	30 38997 8481	4 33969 00551 0	43 36004 459	866 37783			
08	34 72966 8536	4 33925 64546 6	43 35138 081				
09	39 06892 4991	4 33882 29408 5					
10010	43 40774 7932						

There we find that the sixth difference is rapidly becoming less; at the top of the page it is 5202, and at the end of the 200 operations it should shrink to 4619. Now the third class computer, to whom the ruled sheet with only the first line inscribed on it was delivered, must necessarily have carried the same sixth difference all the way down, and therefore, even on the most favourable supposition that all the differences had been carried out to the twenty-sixth place, the result must have been egregiously erroneous.

Again, we come inevitably to the number 10010. Now the essential character of the denary system of logarithms, that from which it derives all its advantages, is this,—that the mantissa of the logarithm of such a number as 10010 is an exact copy of that for 1001; but this number 1001 has had its logarithm already computed to nineteen places, so that we have only to collate the two in order to verify this much of the work. That is to say, this great unwieldy gap of 200 intervals had already been divided into twenty smaller gaps, and the labour of the interpolation has been uselessly augmented at least one hundred times.

And yet further, in the accompanying second scheme I have put the logarithms of the first ten numbers on the first sheet of the Cadastre manuscript, true all to the fourteenth place, with their differences of the first, second, and third orders. Differences of the fourth order only make their appearance in the sixteenth decimal place, and are here awanting. Fourteen place logarithms, then, for numbers above 10,000 can have no differences of the fourth, fifth, and sixth orders; and all this display of high orders of differences and of additional places of decimals has been a matter of pure supererogation.

N.	Log.	1st Diff.	2d.	3d.
10000	·00000 00000 0000	4 34272 7686	43 4207	86
01	04 34272 7686	4 34229 3479	43 4121	86
02	08 68502 1165	4 34185 9358	43 4035	89
03	13 02688 0523	4 34142 5323	43 3946	85
04	17 36830 5846	4 34099 1377	43 3861	87
10005	21 70929 7223	4 34055 7516	43 3774	87
06	26 04985 4739	4 34012 3742	43 3687	87
07	30 38997 8481	4 33969 0055	43 3600	86
08	34 72966 8536	4 33925 6455	43 3514	87
09	39 06892 4991	4 33882 2941	43 3427	86
10010	43 40774 7932	4 33838 9514	43 3341	
11	47 74613 7446	4 33795 6173		
12	52 08409 3619			

Here the third differences are, as it were, constant; the irregularities shown by them are, as every calculator knows, due to the neglect of the farther decimal parts. The third differences, if absolutely true, should show a slight steady diminution, and thus we may expect, without any calculation, that the two succeeding third differences should be from 88 to 85. Now the logarithm of 5006 had been computed to nineteen places, and if we write the logarithm of 2 on the edge of a piece of paper, and then appose it to that of 5006, we shall obtain at a glance the logarithm of 10012. However, we shall not even take the trouble of this addition, but shall content ourselves with the fourteenth figure, which we find to be 9. Let us write this 9 in its proper place, which is two lines below. All that remains for us is to discover what two digits must be written in the last place of the third difference in order to produce this digit 9, knowing also that these digits should be about 6.

The strict investigation is simplicity itself. Let us put a, b, c, d, for the last numbers in the successive columns a_1, b_1, c_1, d_1, for those which are to succeed a_2, b_2, c_2, d_2, for the next again, and we find

$$a_2 - a - 2b + 3c = 3d_1 + d_2 .$$

In resolving this equation we need attend only to the last figures, so that, applying it to the example before us,

$$\begin{aligned} \cdots 9 - 2 - 2 + 2 &= 3d_1 + d_2, \text{ or} \\ \cdots 7 \phantom{{}- 2 - 2 + 2} &= 3d_1 + d_2 . \end{aligned}$$

Recollecting that d_1 and d_2 must be absolutely or nearly alike, we may put for trial

$$\cdots\cdots 7 = 4\, d_1,$$

whence d_1 is nearly 7, and we have actually $d_1 = 7$, $d_2 = 6$; so that the succeeding pair of differences must be 87, 86. Writing these in their places, we readily compute the next pair of logarithms as shown by the slender figures.

By a computation performed mentally in a few seconds, without putting pen to paper, we thus compute the third differences for the next pair of logarithms, with the certainty that the alternate one is absolutely true in the last place, and with a very slight uncertainty as to the intermediate. The whole magnificent array of fourth, fifth, and sixth differences, with decimals to the twenty-sixth place, disappears.

This amounts to the recognition of the fact that the large gap of 200 intervals had already been supplied with 100 stepping-stones, a fact of which neither Prony nor any one of his regiment of geometers seems to have been aware. The method followed in the calculation of the Cadastre table of logarithms was an egregious blunder. The result was in accordance with the method.

After having told us that, for numbers above 10,000, the errors in Brigg's last three figures are not given, M. Lefort goes on to say (page 997):—“For all that there do exist numerous divergencies in the thirteenth and fourteenth places. We must thence conclude that the calculators did not suppose the Cadastre tables to have enough of precision for correcting Brigg's last places. In that they were perfectly right. The bases of the calculation had been chosen so as to make sure of twelve places, and the precautions taken were only applicable to the research of these twelve places, which are all that are fit for publication.”

In order, after the method, but not behind it in importance, comes the conscientiousness with which the work is performed.

The careful computer who may have to revise his own work puts the first performance aside, even leaves it for a considerable time, lest the sight of the figures, or the remembrance of them, should lead to the repetition of an error: better still, he arranges the operation in another way, so that the same additions and subtractions may not recur. But when he has to do with hired assistants he must contrive safeguards against carelessness, even against simulation. Against the former it is possible, though difficult, to protect one's self, but against the latter there is no protection other than in repeating the work, which comes, in effect, to the dismissal of the delinquent. The most obvious safeguard is to have the work done by two or more computers who have no opportunity of intercommunication.

Now the above described fundamental arrangement of the work excluded the possibility of such isolation. The computer—a very ordinary computer, knowing little of mathematics—was desired to make a chain of calculations involving about 13,000 figures; and the test, to him, of the accuracy of his performance was to be found at its conclusion. By the time that the twentieth part of this task was done, the calculator must have felt the danger of error; perhaps he revised thus far on loose paper, found errors, and corrected them. Another division of the task was done in the same way, and, with trembling, the conscientious worker having toiled for several days, arrived at the end, to find, in spite of all his care, some lamentable errors. The search for the source of these, and their correction, might occupy more time and be more irksome than the actual work. But his collaborator had experienced the same troubles. In order to lessen these, and really to improve the performance, the two came to exchange their loose sheets; and the habit of working on separate papers from which copies were made upon the official sheets came to be firmly established. This was inevitable, and might easily have been foreseen.

On page 998, M. Lefort details circumstances which seem to him to prove that the majority, if not the totality, of the calculators of the third section made their additions and subtractions on loose leaves, which they could dispose of freely; afterwards writing the results on the ruled official paper. He remarks, "This was incontestably a fault." It was, in my opinion, a complete relinquishment of the safe-guard afforded by duplicate manipulation. Now this gross and habitual infraction of the official rules could not have remained hid from the superintendents. It must have been winked at; nay, it must have been controlled by them. The existence of many local and restricted "surcharges," or corrections, proves that the loose paper of the one had been collated with that of the other computer; and thus the whole operation was conducted with a laxity of discipline which detracts enormously from its value.

But M. Lefort tells us, on page 996, that Prony was so jealous of the errors induced by transcription, that, when pressed by the Government to extract seven-place trigonometric tables from his extended ones, he preferred to proceed to their direct construction, rather than to incur the risk of the errors of copying. To me this appears an unintelligible motive; because, whether computed directly or not, the table must be copied in type, while the probability of exactitude is immensely in favour of the extended calculation. Lefort proceeds to say, "On no occasion did Prony or his collaborators say or give reason to think that there had been copying on the ruled sheets."

It is to be remarked, that Lefort does not advert at all to that very circumstance which gave occasion for his paper, namely, the presentation to the Library of the Institute of a third copy of the great tables: yet he says, on page 995, that the collection of the leaves filled by interpolation was to form a double original; that such was the object, the only object of the operation, "Tel ètait le but, le but unique de l'operation." On what hypothesis, then, shall we explain the existence of this third copy "qui avait été laissé à Prony à titre de Minute." M. Prony could not have possessed it and been ignorant of the fact that at least one of the three was the result of transcription. Yet for these three quarters of a century the whole mathematical world has been led to believe, has believed, has acted, and has spoken in the belief that these "Grandes Tables" were constructed with every, the most scrupulous, attention to the requirements of exactitude.

Having made a most careful examination of the logarithmic part of the work—having performed the duty of a most conscientious witness in stating the facts as they appeared to him—M. Lefort has not ventured to sum up the evidence; but, speaking of the tables, concludes his paper with the words, "Je n'ai voulu aujourd'hui qu'en constater la valeur," leaving to others to form their own opinions of the exact value determined by his revelations.

He does, indeed, express a qualified commendation, for, at the foot of page 998, he says:—"The Cadastre Tables, like all human works, are then not perfect; they are so neither in their execution nor, perhaps, in the details of their conception; nevertheless, they much surpass, not only in extent, but also and above all in correctness, all the tables that have preceded them, and the more modern tables which have not been compared with them before publication."

Here M. Lefort has omitted to observe that he had been collating a manuscript calculation, in which there should have been no error, with printed books subject to all the chances of mistakes in reading and accidents in printing; he has surely also forgotten that these manuscript tables were so imperfect, and were known to their computers to be so imperfect as to be unfit for the verification of the last three places in the "Arithmetica Logarithmica," the very first work on denary logarithms, a work undertaken and completed by a private person, amid all the difficulties and round-aboutness of infant algebra. As to the details of the conception, he has told us that the orders of differences were extended to the sixth, that the decimal places went to the twenty-sixth, with the admirable result of an exactitude not reaching beyond the twelfth place, where differences of even the third order barely appear. And above all, he has failed to perceive that what confidence we can now have in these prodigious piles of figures is derived from the labours of a single individual, whose zeal and perseverance led him to collate with the Cadastre table, the only two tables which preceded them, and to examine the divergences by help of calculations more trustworthy than either; that, in fact, the portion of the "Grande Tables" entitled to claim our confidence rests that claim on the joint labours of Adrian Vlacq and M. F. Lefort. The further *dictum*, that these tables are more exact than later ones which have not been compared with them, is supported by no evidence or argument, besides implying an obvious absurdity.

Concerning the rest of the Logarithmic Table, that belonging to numbers from 100,000 to 200,000 we have no information, because there existed no table for comparison, and our confidence must be founded exclusively on what we know of the principles of the method followed, of the fidelity of the execution, and of the candour of the statements. On these three heads, M. Lefort has placed before us information which it is not necessary for me to recapitulate.

The advantages of a uniform scale of numeration have been recognised in all ages. The ancient geometers adopted the basis 60, and their system has come down even to the present day. If the circumference of the earth be divided into 60 parts, each of these again into 60, and, once more, each of these subdivisions into 60 parts, we come almost exactly to the stadium; according to the same plan, the hour and the degree are each divided in sixtieths. It is the uniformity, the self-consistency of this system, which has so long preserved it. Twenty centuries ago, the sage of Syracuse placed before King Gelo the powers of the more convenient denary system; he showed how a few steps of a progression by myriads enabled him to express the number of grains of sand, not in the bay of Syracuse, not on the whole shores of Sicily, but that would be contained in a sphere having the moon's distance for its radius. The denary system has gradually gained supremacy in the languages of all civilised nations, to the obliteration, in most of them, of every trace of any other system; 260 years ago, it was crowned by the invention of logarithms, which invention has rendered its exclusive use in every department of science only a question of time. Briggs computed a trigonometric table to each hundredth part of the degree; and near the end of the last century the still further improvement was proposed of dividing the quadrant itself into one hundred degrees, and each degree centesimally. Borda, about 1793, computed, to each minute of this division, a table, which was given in a compressed and inconvenient form by Callet, in his "Tables Portatives;" but an extension of this, at least so far as to each tenth second, was absolutely needed before the centesimal division could be used in the higher departments of geodesy and astronomy.

The French government, with a most enlightened regard for the interests of science, that is, for the interests of humanity, ordered the computation to be proceeded with, and entrusted the execution thereof to M. Prony. In confident expectation of the speedy appearance of this table, M. de La Place gave in his great treatise on Astronomy, the "Mécanique Celeste," the data and formulæ, according to the new division of the circle. The calculations have been finished, but have remained in the libraries of the Institute, and of the Observatory, inaccessible and useless to the general scientific public.

It would be difficult indeed to over-estimate the injury done to the progress of exact science by this calculation and this occlusion. accompanied, as they have been, by the pretence of exhaustive accuracy and unrivalled extent. There was enough of enterprise, enough of zeal, to have long ago completed the necessary work to seven places. Michael Taylor, in 1792, had finished a far more extensive work, the compilation of a seven-place table to each single second of the old division. But these existent and unpublished tables barred the way; for no private person would think of undertaking of new a work which had been already so well accomplished. Thus the most excellent and most laudable design of the French Government has been frustrated,—has been turned from a benefit to an injury.

Though sorely needed and urgently demanded, the new tables did not appear; and when expectation had been stretched to the utmost, the English Government, in 1819, at the instance of Mr Davies Gilbert, proposed to defray one-half of the expense. The negociations led to no result. M. Lefort gives an extract from a note addressed by the celebrated astronomer Delambre to the English commissioner, apparently the farewell note. I transcribe the extract from page 999.

" Ces tables, non plus que celles de Briggs, ne serviront pas dans les cas usuels, mais seulement dans des cas extraordinaires. Comme celles de Briggs, elles seront la source où viendrous puiser tous ceux qui impriment les tables usuelles avec plus ou moin d'étendue. Elles serviront de point de comparaison pour tout ce qui a été fait ou se fera."

Whether shall we accept this magniloquent praise or the refusal to print the tables as the measure of their value? Even had these tables been all which they should have been,—all that was pretended for them,—the concluding sentence is preposterous. Is every calculation in all futurity to be tested by comparison with Prony? No! Even away from the revelations of M. Lefort, the independent original computer would not seek to dip his pitcher in the well at the Bureau de Cadastre, he only cares to fill his cup at the small everflowing spring of conscientious performance.

The tables of Prony cannot be printed without entire revision; in such a case to revise is to supersede, and therefore I call upon the whole body of cultivators of exact science to shake off this incubus, to hold these tables as non-existent, and to face manfully the problem of computing decimal Trigonometrical Tables of extent and precision sufficient for their pioneers, and therefore capable of supplying all the shorter and less precise tables needed for their more ordinary pursuits.

2. On the Elimination of α, β, γ, from the conditions of integrability of $S. u\alpha\delta\rho$, $S. u\beta\delta\rho$, $S. u\gamma\delta\rho$. By M. G. Plarr. Communicated by Professor Tait.

3. The Development of the Ova, and the Structure of the Ovary, in Man and other Mammals. By James Foulis, M.D. (Edin.) Communicated by Prof. Turner.

After an historical introduction, in the course of which the author gave an abstract of the important observations of Pflüger and Waldeyer, he proceeded to state his own observations on the development of the ova and structure of the ovary in calves, kittens, and the human female. The following general conclusions have been arrived at by the author in the course of his investigations:—

The corpuscles of the germ epithelium are derived by direct proliferation from those columnar corpuscles which invest the median side or surface of the Wolffian body, and which are continuous with the layer of columnar corpuscles that lines the pleuro-peritoneal cavity of the embryo in the early stages of development. The stroma of the ovary in the early stages of development is produced by a direct growth out from the interstitial tissue of the Wolffian body immediately beneath the germ epithelium on the median side of the Wolffian body.

The germ epithelial corpuscles proliferate by fission. In the human fœtal ovary of 7½ months they measure $\frac{1}{2500}-\frac{1}{2000}$ of an inch in their longest diameter, and about $\frac{1}{3500}$ of an inch in their shortest diameter. Each germ epithelial corpuscle is a nucleus surrounded by a thin film or investment of clear protoplasm. The nucleus of each germ epithelial corpuscle becomes the germinal vesicle of the mature ovum; and every germ epithelial corpuscle is potentially an ovum. In the act of becoming primordial ova, the nucleus of each germ epithelial corpuscle swells up into a spherical corpuscle with dark granular contents, within which is generally seen a nucleolus, and around which is produced clear homogenous protoplasm which subsequently forms the yelk of the ovum. Germ epithelial corpuscles are seen in all stages of development into primordial ova. In each primordial ovum the spherical germinal vesicle presents a sharply defined limiting membranous wall. Within the germinal vesicle is the nucleolus or germinal spot. All the ova in the ovary are derived from germ epithelial corpuscles.

In all parts of the ovary processes of vascular connective tissue stroma grow in, between and around certain of the germ epithelial corpuscles, whereby the latter become more and more embedded in the stroma of the ovary. Germ epithelial corpuscles are being constantly produced on the surface of the ovary, to take the place of those already embedded in the stroma. The embedded corpuscles increase in number by division, and the nucleus of each swells up into a spherical germinal vesicle, around which is gradually produced the yelk of the ovum. In all parts of the young ovary under the germ epithelium, groups of germ epithelial corpuscles become embedded in meshes of the stroma. As each individual in the group swells up the nucleus or germinal vesicle of each becomes very distinct as a round or spherical body. From the swelling out of each germ epithelial corpuscle in the group, the whole group expands and becomes more or less spherical. Such groups of developing corpuscles are called egg clusters. Each egg cluster is enclosed in a mesh or capsule of vascular stroma of the ovary.

The stroma of the young ovary consists for the most part of fusiform connective tissue corpuscles and blood-vessels. The walls of the young blood-vessels in the young stroma consist of connective tissue corpuscles. These connective tissue corpuscles are direct offshoots from the ovarian stroma, and are found in contact with the yelk or protoplasm of each primordial ovum situated among the germ epithelial corpuscles on the surface of the ovary. Wherever we find primordial ova we see connective tissue corpuscles in contact with the yelk of each. In all parts of the ovary we find the nuclei of connective tissue corpuscles dividing. Sometimes these corpuscles are swollen out into round bodies containing three to four nuclei. In each egg cluster several of the included germ epithelial corpuscles are in a much farther advanced stage of development than their fellows. From the walls of the meshes enclosing the egg clusters, delicate processes of vascular connective tissue grow in, between, and around individual corpuscles in the egg clusters, and by a continued intergrowth of the young stroma in this manner each individual of the group becomes at last enclosed in a separate mesh or capsule. These last formed meshes are the Graafian follicles.

As a rule, each Graafian follicle is occupied by one young ovum. The protoplasm or yelk of each ovum is in close contact with the wall of each Graafian follicle. In contact with the yelk of each young ovum, and indenting it, are connective tissue corpuscles, which form part of the wall of each Graafian follicle. In the formation of the membrana granulosa, these connective tissue corpuscles in the wall of the Graafian follicle, and in contact with the yelk of the contained ovum, increase in number by division, their nuclei swell out into little vesicles, and at last a perfect capsule of such corpuscles is produced round the ovum. This capsule is the membrana granulosa or follicular epithelium of the follicle. At first the membrana granulosa consists of a simple layer of cells lining the follicle. The individual corpuscles of the membrana granulosa measure about $\frac{1}{3000}$ inch. As the ovum becomes mature, the corpuscles of the membrana granulosa proliferate, and then many layers of small corpuscles are produced between the ovum and the follicular wall. The cells of the membrana granulosa are thus derived from the corpuscles of the connective tissue stroma, and not, as Waldeyer states, from the germ epithelial corpuscles. The follicular space is formed by a breaking down and probable solution of certain of the corpuscles of the thickened follicular epithelium in the middle parts of the same. The discus proligerus consists of follicular epithelial corpuscles, which are in contact with the zona pellucida of the ovum. The zona pellucida or vitelline membrane is formed by a hardening of the outer part of the yelk or protoplasm of the ovum, and is not, as Reichert, Pflüger, and Waldeyer stated, a product of the follicular epithelium. At birth the human ovary contains not less than 30,000 ova, few of which reach maturity.

In the human ovary at birth the germinal vesicles measure $\frac{1}{1500}-\frac{1}{1200}$ of an inch. Most of them are about the same size, and also present a sharply-defined membranous wall. In some germinal vesicles two or three germinal spots are seen. The tunica albujinea is the thickened stroma growing round the ovary. At the age of 2½ years all formation of ova from the germ epithelium has ceased.

Graafian follicles are not formed from tubular structures in the manner described by Pflüger, Spiegelberg, and Waldeyer. The appearances of tubular structures passing into the stroma of the ovary are produced by sections through furrows and depressions between irregular prominences on the surface of the fœtal ovary. The irregularities of the surface of the fœtal ovary are produced by the expansion of egg clusters upwards under the germ epithelium. When the walls of furrows and depressions come in contact, egg clusters are formed by the embedding of germ epithelial corpuscles in that situation, just as in other situations. Egg clusters are formed in connection with the germ epithelium lining the furrows and depressions. Among the germ epithelium corpuscles lining the furrows, &c., we find large primordial ova, and corpuscles in all stages of development into the same, just as in other situations among the ordinary germ epithelial corpuscles.

At the age of six years the epithelium on the human ovary consists of very small flat hexagonal-shaped corpuscles, measuring $\frac{1}{2800}-\frac{1}{3200}$ of an inch. The corpuscles are seen dividing. This layer can be stripped off without difficulty. At the age of twelve the epithelium has little difference in appearance from the above, the small size of the epithelial corpuscles being remarkable. The epithelium is beautifully seen in old cats, and must be regarded as homologous with the peritoneal epithelium. In old cats the epithelium on the surface of the ovary consists of very small distinct cells, measuring from $\frac{1}{1800}$th to $\frac{1}{2000}$th inch, with granular oval nuclei.

4. Mathematical Notes. By Professor Tait.

(*Abstract.*)

(1.) On a singular Theorem given by Abel.

The theorem in question, in its simplest form, is

$$\pi f(x)=\int_0^x \frac{dy}{\sqrt{x-y}}\int_0^y \frac{f'(\xi)d\xi}{\sqrt{y-\xi}}.$$

Abel's proof of it involves the properties of the gamma-function, and requires that $f'(\xi)$ should be capable of development in powers of ξ. (*Œuvres*, I. 27.)

Independently of the interesting kinetic application for which it was originally designed, this result is very curious, as suggesting a form of the *square root* of the operation of simple integration. In fact it gives

$$\left(\frac{d}{dx}\right)^{-\frac{1}{2}}(\ \ ) = \frac{1}{\sqrt{\pi}}\int_0^x \frac{(\ \ )dy}{\sqrt{x-y}}.$$

Seeking to obtain an elementary proof of Abel's result, which should at the same time be applicable to any function, whether developable or not, I hit upon the simple expedient of inverting the order of the two integrations. We thus get the proof immediately in the form

$$\int_0^x \int_0^y \frac{dy\, f'(\xi)d\xi}{\sqrt{x-y}\sqrt{y-\xi}} = \int_0^x \int_\xi^x \frac{f'(\xi)d\xi dy}{\sqrt{x-y}\sqrt{y-\xi}}.$$

Now it is known (and a simple geometrical proof is easily given) that

$$\int_\xi^x \frac{dy}{\sqrt{x-y}\sqrt{y-\xi}} = \pi.$$

Hence the integral becomes at once

$$\pi\left[f(x) - f(0)\right].$$

Numerous extensions and applications of the theorem are given.

As one example of these extensions the following, which assigns an expression for $\left(\int_0^x\right)^{\frac{1}{n}}$, may here be given—

$$\int_0^{x_1} \frac{dx_2}{(x_1 - x_2)^{\frac{n-1}{n}}} \int_0^{x_2} \frac{dx_3}{(x_2 - x_3)^{\frac{n-1}{n}}} \cdots\cdots \int_0^{x_n} \frac{f'(\xi)d\xi}{(x_n - \xi)^{\frac{n-1}{n}}}$$

$$= E_1 E_2 \ldots\ldots E_{n-1}\left[f(x_1) - f(0)\right].$$

Here

$$E_r = \int_0^1 \frac{de}{(1-e)^{\frac{n-1}{n}}\, e^{\frac{n-r}{n}}},$$

and therefore

$$E_1E_2\ldots E_{n-1} = \frac{\Gamma\left(\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{\Gamma\left(\frac{2}{n}\right)} \cdot \frac{\Gamma\left(\frac{1}{n}\right)\Gamma\left(\frac{2}{n}\right)}{\Gamma\left(\frac{3}{n}\right)} \cdots \frac{\Gamma\left(\frac{1}{n}\right)\Gamma\left(\frac{n-1}{n}\right)}{\Gamma(1)} = \left(\Gamma\left(\frac{1}{n}\right)\right)^n.$$

Hence

$$\left(\frac{d}{dx}\right)^{-\frac{1}{n}}(\quad) = \frac{1}{\Gamma\left(\frac{1}{n}\right)} \int_0^x \frac{(\quad)dy}{(x-y)^{\frac{n-1}{n}}}.$$

The theorem given by Abel is easily seen to be the particular case of this when $n = 2$, for then

$$\left(\Gamma\left(\frac{1}{2}\right)\right)^2 = \pi.$$

Another form of the above multiple integral is easily seen to be

$$x_1 \int_0^1 \frac{e_1^{\frac{n-1}{n}} de_1}{(1-e_1)^{\frac{n-1}{n}}} \int_0^1 \frac{e_2^{\frac{n-2}{n}} de_2}{(1-e_2)^{\frac{n-1}{n}}} \cdots\cdots \int_0^1 \frac{f'(e_1e_2 \ldots\ldots e_nx_1)de_n}{(1-e_n)^{\frac{n-1}{n}}};$$

and curious expressions for $\left(\frac{d}{dx}\right)^{-\frac{2}{n}}$ (when n is even) may be obtained by evaluating the integral

$$\int_0^{x_1} \frac{dx_2}{(x_1-x_2)^{\frac{2(n-1)}{mn}}} \int_0^{x_2} \frac{dx_3}{(x_2-x_3)^{\frac{2(m-1)(n-1)}{mn}}} \cdots\cdots$$

$$\cdots\cdots \int_0^{x_{n-1}} \frac{dx_n}{(x_{n-1}-x_n)^{\frac{2(n-1)}{mn}}} \int_0^{x^n} \frac{f'(\xi)d\xi}{(x_n-\xi)^{\frac{2(m-1)(n-1)}{mn}}}$$

where m is any real quantity whatever.

Other instances of the use of this process were adduced, but those just given are sufficient for an abstract like the present.

(2.) On the Equipotential Surfaces for a Straight Wire.

Some results given in Vol. I. of *Thomson and Tait's Natural Philosophy* may be much more simply obtained by calculating the potential of a wire rather than its attraction. That potential is easily found as

$$\rho \log. \frac{r_1 + r_2 + c}{r_1 + r_2 - c},$$

where c is the length of the wire, ρ its line density, r_1 and r_2 the distances of its ends from the point at which the potential is to be found.

In this form we see at a glance that the equipotential surfaces are prolate ellipsoids of revolution with common foci at the ends of the wire.

The method is extended to polygons plane or gauche, closed or unclosed.

(3.) On a Fundamental Principle in Statics.

The principle that, while additional constraints cannot disturb equilibrium, unnecessary constraints may be removed without disturbing equilibrium, is of very great use in the statics of fluids and of elastic and flexible bodies. But it seems not to have been made use of to the extent its importance deserves.

My attention was recalled to it when attempting to compare the shares taken by gravity and cohesion in resisting the tendency of the so-called centrifugal force to split a planet. The problem which first proposed itself was to determine the gravitation attraction of one-half of a uniform sphere upon the other.

The sextuple integral which a direct solution of this problem would require may be entirely dispensed with, and its place supplied by a simple single integral, if we imagine a thin film of the solid on each side of a diametral plane to be converted (without change of bulk or density) into an incompressible liquid.

Or we may commence with a sphere of homogeneous incompressible liquid. If a be its radius, ρ its density, it is easily shown that the whole pressure normal to any diametral plane—which is of course the attraction of the hemispheres on one another—is

$$\frac{1}{3}\pi^2\rho^2a^4.$$

If each hemisphere were collected at its centre of inertia the attraction would be $\left(\frac{4}{3}\right)^3$ times as great. The centrifugal force tending to split the planet across a diametral plane through the axis (it is easily shown to be greater per unit of area on a diametral than on any other plane) is

$$\frac{1}{4}\pi\rho\omega^2a^4,$$

where ω is the angular velocity of rotation. The ratio of these is

$$\frac{\frac{4}{3}\pi\rho a}{a\omega^2}$$

or the ratio of gravity to centrifugal force at any point on the equator. Hence, so far as gravity is concerned, the earth would split across a meridian if it were to revolve more than seventeen times faster than it does.

It is known that, if the earth revolved seventeen times faster than it does, centrifugal force would just balance gravity at the equator. The relation of this fact to the above statement depends upon the geometrical proposition that the volume of a very small slice from the surface of a sphere is half the product of its thickness by the area of its base.

And cohesion would not sensibly alter this state of things; for, assuming the earth's diameter to be 8000 miles, its mean density 5·5, and the weight of a cubic foot of water at the surface 63 lbs., while the average tensile strength of its materials is taken as 500 lbs. weight per square inch, the cohesion between two hemispheres is shown to be only $\frac{1}{25,410}$th part of their gravitation attraction.

Even if we made the extreme assumption that the tensile strength is (throughout) that of steel, cohesion would in the case of the earth be only about $\frac{1}{100}$th of gravitation attraction between hemispheres.

As a consequence, a planet of the earth's mean density and the above assumed tensile strength is held together as much by cohesion as by gravitation if its radius is $\frac{1}{\sqrt{25,410}}$th of that of the earth, or about 25 miles. If of steel's tenacity it would have a radius of about 409 miles.

Monday, 4th January 1875.

Sir WILLIAM THOMSON, President, in the Chair.

The President exhibited and described his Tide Calculating Machine, also his Improved Tide-Gauge; he also described certain Capillary Phenomena, with Experiments.

The following Gentlemen were elected Fellows of the Society:—

C. H. Millar, Esq.
John Milroy, Esq.
Professor Daniel Wilson, Toronto.
Anderson Kirkwood, LL.D.
Dr Ludwik Bernstein.
Daniel G. Elliot, Esq., New York.
Robert Gray, Esq.
William Craig, M.D., F.R C.S.E.

Monday, 18th January 1875.

The Hon. Lord NEAVES, Vice-President, in the Chair.

The following Obituary Notices of Deceased Fellows of the Society were read:—

1. Biographical Notice of Lord Colonsay. By the Hon. Lord Neaves.

By the death of Lord Colonsay, this Society has lost a member of great distinction, and well worthy of being held in respectful remembrance. He was a man of great vigour of mind, and with powers and qualities which would have earned for him a high place in science or in literature if they had been turned in either of these directions; but it was his lot and his choice to follow a professional career, in which, as will afterwards be seen, he came to attain all the varied honours which the practice of the law is able to confer.

Duncan M'Neill was the second son of John M'Neill, proprietor of the islands of Colonsay and Oronsay, and of the estate of Ardlussa in Jura, and was born in Oronsay on the 20th of August 1793. He was not educated at any school, but received private tuition at home along with his brothers, until he repaired to the University of St Andrews, along with his immediately younger brother, now Sir John M'Neill.

He used in after life to tell of an incident that occurred to the party when his father and the two boys passed a night in Glasgow on their way from the Highlands to St Andrews, and it was certainly one well calculated to make a permanent impression on a vigorous and appreciative mind. While he was walking in the morning, near the post-office, a mail-coach arrived, from the roof of which the guard announced to an assembled multitude the news of the victory of Trafalgar, which occurred on 21st October 1805. The intelligence, of course, was received with tumultuous cheering, after which, one of the crowd proposed three cheers for Nelson, but when the guard in a loud and sad whisper said "that Nelson was killed," they all instantly dispersed in solemn silence, and left the streets empty.

Duncan M'Neill was twelve years old when he went to St Andrews, which was not at that time an unusual age for college intrants. He and his brother were boarded with Dr James Hunter, professor of Logic, for whom and for his family M'Neill always entertained a strong feeling of attachment and regard. He became a diligent student and a good classical scholar, but was still more distinguished in mathematics, for which he had a remarkable aptitude.

After three years spent at St Andrews he came to Edinburgh, and attended college here for some sessions. As usual with young men of intellectual power, he applied himself diligently to Logic and Metaphysics, for the latter of which, undoubtedly, Dr Thomas Brown, whom he attended, was calculated to inspire a strong taste, though Brown himself was not a profound or perhaps even a sound metaphysician. His lectures, however, were pleasing and attractive, and had often the effect of leading his hearers to larger and deeper views than their teacher entertained.

It has been stated in a very able and kindly notice of Lord Colonsay, ascribed to Mr Campbell Smith, that about this period, or shortly afterwards, M'Neill formed the intention of publishing the philosophical works of David Hume, of which at that time no good collective edition existed. I am not able to confirm this statement by my own testimony, but I know well that he was always a great admirer of our greatest Scottish philosopher. He was not likely to be led away into those sceptical speculations which Hume propounded from his attempting to introduce metaphysics into a region which lies above their reach, nor was he likely to follow Hume in the perverse preference which he seemed to feel for French literature over English, and which may be traced partly to the influence of prejudice, and partly to a feeling that he was less appreciated in England than on the other side of the channel. But in other respects the mind and style of Hume were well calculated to please and influence M'Neill in matters of reasoning and of philosophy. The simplicity and brevity with which he wrote, the caution and moderation with which he stated his opinions, and the calmness with which he dealt with his adversaries, were all congenial to the tastes and feelings with which M'Neill was wont to approach questions of evidence and reasoning. It is not to be supposed that he was destitute of feelings and energies to which Hume was a stranger. His Highland or Island blood was more fervid than any that circulated in Hume's veins, and his early life and athletic frame were a strong contrast to the indolent and somewhat obese form of the philosopher of the Merse.

With a view to a professional life, M'Neill entered on an apprenticeship in the chambers of Mr Michael Linning, W.S., and discharged with regularity and diligence the duties that there devolved upon him.

I am not sure whether it was originally intended by his friends that M'Neill should come to the bar, or whether the remarkable talents which he soon displayed led to his adopting that profession instead of that of a writer to the Signet, to which his initiation at Mr Linning's would naturally have led. But it cannot be doubted that the time passed and the instruction received by him in a writer's chambers, as well as the friendships which he there formed, were eminently useful to him at the bar.

Lord Colonsay passed advocate in 1816, and amidst a great number of eminent contemporaries and rivals he soon became distinguished in his profession. He devoted himself with special diligence to the study of criminal law, which he thoroughly mastered, and made himself so formidable as an opponent in defending prisoners that the Crown authorities saw the advantage of securing his services on their side, and in 1820 he was made an Advocate-depute by Lord Meadowbank, then Lord Advocate.

In 1822 he was appointed Sheriff of Perth, in room of Lord Medwyn, promoted to the Bench. He continued in that office with great efficiency and usefulness down to 1834, when he became Solicitor-General under Sir Robert Peel's administration. That ministry retained office for only a few months; but when they returned to power in 1841, he was again made Solicitor-General. In October 1842 Sir William Rae, then Lord-Advocate, died, and M'Neill succeeded him in that office. In 1843 he was elected Dean of the Faculty of Advocates, and became Member of Parliament for Argyllshire, holding that position from 1843 to 1851, when he was promoted to the Bench by the Whig Ministry at the same time with Lord Rutherfurd. In 1852 he was made Lord Justice-General and President of the Court of Session. After serving in that high position for fifteen years he was created Baron Colonsay in 1867, when he retired from the bench.

Thus it is that Lord Colonsay passed through all the grades and honours of his profession, from that of a simple advocate to the Presidency of the Court. We do not know if this is unprecedented, but it certainly has rarely happened that a member of the bar has become successively, as Lord Colonsay did, a Depute-Advocate, a Sheriff, Solicitor-General, Lord Advocate, Dean of Faculty, an ordinary Judge, and finally Lord Justice-General and Lord President. The varied functions and wide experience which these successive positions involved, could not fail to qualify him in the highest degree for the discharge of all his duties, and above all, of those which ultimately devolved upon him when placed at the head of legal administration of Scotland. Every professional man knows that the inferior grades of legal preferment are eminently conducive to furnish the necessary knowledge and practice required for higher positions. It cannot be doubted that great experience as an advocate at the bar is of the highest use in discharging the functions of the bench. Under some national systems, Judges have been chosen who had not practised as advocates, but they would certainly not possess in that way the intelligence and penetration which an experienced barrister acquires, and which must enable him when on the bench to weigh the evidence, to detect the truth, and to see quickly through the fallacies and disguises to which litigants are apt to resort. In another way the exercise of the inferior jurisdiction of Sheriff brings the holder of office into closer contact with country matters, and with local and customary considerations, which will serve him in good stead when as a Judge he comes to sit in review upon County-Court procedure.

Lord Colonsay was every way qualified for the profession which he adopted, and for the offices which he held. His talents, which were great, were eminently of a forensic and still more of a judicial character. His logical acumen was severe and unerring. He possessed also, though he never exercised it unnecessarily, a power of vivid and impressive eloquence, in which he was equalled by few and surpassed by none. He was a most able criminal advocate, and indisputably the greatest criminal lawyer of his day. His natural powers were aided and improved by patient and laborious study as a young man, and by the most conscientious and careful discharge of duty in all matters that came before him, whether at the bar or on the bench. Those who had the advantage of meeting him in consultation as an advocate, will bear testimony to the thorough mastery which he always attained of his client's case, and to the sagacious and skilful perception which he also acquired of the probable case of his opponent. In consultation he was entirely free from the petty selfishness that has sometimes been laid to the charge of seniors in bottling up their best views for their own use. Whatever point he thought advantageous to the case was always fully communicated and explained to his juniors.

In the practice of his profession as an advocate Lord Colonsay had some advantages not equally enjoyed by some of his brethren. The subjects with which an advocate has to deal are so various, and often so special and technical, that it is impossible for any man to have a thorough and independent knowledge of all. The advocate has what may be called a *nisi prius* faculty of learning, on short notice, what he knew nothing of before, and then forgetting it when the occasion is over, in order to make room for new acquisitions equally temporary and transient. His great art consists in knowing where information is to be found, and making the appropriate use of it for his immediate purpose. In an extensive practice an advocate is thus brought in contact with questions of the most dissimilar kind—commerce, agriculture, engineering, chemistry, and many others, arising out of multifarious patents or contracts that become the subject of litigation. I once was able to illustrate this somewhat oddly to a man who knew many subjects and wrote many books. The late Mr MacCulloch, the political economist, once asked me in company whether his "Commercial Dictionary," which is a very useful book, was ever founded on or quoted in our courts of law? I answered rather abruptly, "Never; the name of it is never heard." He appeared disappointed at this, and I then added, "But very often a case comes in to us at night to prepare for next day, on a subject we know nothing about—general or particular average, foreign exchanges, or the like—upon which we go to our shelves and take down a Commercial dictionary, which enables us to appear at the bar when wanted next day with an amount of information that astonishes even our own clients. But we never mention the book from which the information is got." This statement seemed completely to re-establish the self-complacency of the sensitive author.

I would say here that Lord Colonsay, from his scientific tastes and tendencies, was more fully and accurately grounded in many of these questions than the most of his brethren. And this could not fail both to lighten his labours and to give confidence to his views.

As a judge, his judgments were models of clearness and brevity, and were always remarkable for an anxiety to maintain the great landmarks of legal principle. If he had a fault, it was one which, I think, in judicial business, "leans to virtue's side." When he felt that he could not be bold he was apt to be very cautious, and certainly was ever anxious not to decide any case but the one that was immediately before the Court, leaving other cases to be determined at their own time, and after fully hearing the arguments that were specially directed to discuss them; and I am much inclined to think that it is better always to decide nothing but the actual question raised, or necessary to be decided, as no collateral point can in general receive the mature treatment and consideration that it deserves. His perfect command of temper, his great patience in listening, and his uniform courtesy on the bench earned for him the respect and gratitude of the whole bar, and added greatly to the weight and authority of his judgments.

We should not fully do justice to Lord Colonsay's merits if we did not notice and acknowledge the important benefits which the country has derived from his legislative exertions. At an early period, I believe, we may say that the great improvements made on criminal procedure in Scotland, by an Act in the ninth year of George IV., emanated from Lord Colonsay, though Sir William Rae was at that time Lord Advocate. The older forms of criminal process in Scotland, whatever may have been said to the contrary, were highly, and perhaps unduly, favourable to accused persons—in this respect, at least, that many formal objections to the designation and citation of witnesses and otherwise could be kept back till after a jury was empanelled, and could then be brought forward so as to frustrate the proceedings, while at the same time the accused could not be tried again in consequence of having "tholed an assize." This state of things, of which no one could make a better use than Lord Colonsay when defending prisoners, was abolished; so that all formal objections must now be brought forward at once before empanelling a jury, and thus, even if they prove fatal, the accused can be tried again on a new indictment.

When in Parliament as Lord Advocate, Lord Colonsay passed, or assisted in passing, many useful measures; but perhaps the most conspicuous of these is the Poor Law Amendment Act—a wise and beneficent measure, which has gone far to solve the great social difficulty of relieving pauperism without paralysing industry or oppressing ratepayers, many of whom must always be nearly as poor as the objects who obtain relief.

In all matters of legal reform, Lord Colonsay's services have always been at the command of his country, and though unostentatiously performed, have been thoroughly appreciated by those who had the means of knowing and the power of judging.

Of the debt which we owed to Lord Colonsay after he took his seat in the House of Lords, it is unnecessary to speak.

I may here advert to a part of Lord Colonsay's life which possesses much interest, and is calculated to throw a strong light upon his character. Some time after the death of his father he became by a family arrangement the proprietor of Colonsay and Oronsay, which he retained till a comparatively recent period. In consequence of the advanced age of his father, these estates had not latterly been administered with as much energy and enterprise as the times demanded. They were all in the hands of the proprietor, except some small possessions held by a number of crofters and cotters. When Lord Colonsay acquired the property, he applied himself vigorously to putting it into perfect order. Besides visiting it during the vacations of the Court, he personally directed the whole improvements which were made upon it, and for that purpose transmitted, in the midst of the labours of his profession, minute directions weekly to his managers on the spot, and received their detailed reports of everything that was doing. In a few years he had the islands put into a most satisfactory state for being let out in separate farms of suitable size. The stock on the farms was every way improved. He encouraged and liberally aided emigration, and did so with singular delicacy, so as to spare the feelings and not impair the means of the emigrants. Excellent farm houses and offices were built, roads formed, and harbours improved at a very great expense, and at last he succeeded in lightening his own labours and establishing in the islands respectable tenants whose occupations gradually increased in value. He also succeeded in getting Colonsay detached from Jura and made a separate parish; and having improved the church that had been there in use, and built a comfortable manse and good school, he settled a liberal endowment on the minister, and thus gave the people on the island the advantages of a regular and efficient ministry, and two good parish schools. It may gladden our friend Professor Blackie's heart to hear that he retained his Gaelic in perfection to the last, and was thus enabled to exercise an influence that might otherwise have been lost.

I shall add only a few more words as to his personal life. He was never married, but his younger brother Archibald, with his wife and family, were for many years domesticated with him, and when his brother died, the widow and surviving children remained with him as before, and ultimately shared in a large portion of his means. He was a most affectionate relative, and a very firm friend. He never forgot a kindness received, and had particular pleasure in repaying, when it came to be in his power, any proofs of friendship which he had received in the earlier period of his career, when encouragement and assistance were calculated to be of such value. He was a man of great goodness of temper, and of inflexible justice in all his dealings. His estate of Colonsay he had disposed of before his death to his brother Sir John M'Neill, under a family arrangement.

For a considerable part of his life Lord Colonsay laboured under some weakness in the chest and breathing tubes, and latterly a tendency to bronchitis was perceptible. We believe it was to this malady that he fell a victim. He was only ill for a short time, and at the age of eighty it was not wonderful that he was unable to resist the influence of a disease so dangerous in general to those advanced in life.

2. Biographical Notice of Cosmo Innes. By the Hon. Lord Neaves.

We have lost another eminent member of our Society in Mr Cosmo Innes, of whom I shall venture to give a short account. I do not think it necessary to make it long, and this for various reasons. Mr Innes's labours were more nearly akin to the studies of another Society which meets under the same roof with ourselves, and within that body, I believe, tributes have been paid to his memory far more intelligent and more worthy of his reputation than any I could venture to offer. The general features of his career, also, are so well and widely known, and have been recalled to our recollection of late in such various ways, that any detailed narrative would be superfluous. My endeavour now, therefore, will mainly be not to pay homage to his antiquarian attainments, which are indisputable, nor to the works of interest and utility which have proceeded from his industry, and which are never likely to be forgotten or to remain unappreciated, but to bear my testimony to his general accomplishments, and to his high personal character. Of these I claim a right to speak, from an unbroken friendship of upwards of sixty years, varied by much vicissitude of events, much community of favourite studies, constant professional or official intercourse, and domestic familiarity of the warmest and most pleasing kind.

Mr Innes was born on 9th September 1798. He was educated at the High School of Edinburgh, and at the University of Glasgow, from which last he proceeded on a Snell exhibition to Balliol College, Oxford.

It is well known, and necessary to be remembered, that the position of Mr Innes's family while he was yet a young man, came to be greatly affected by a misfortune that befell his father. Mr Innes, senior, who was a Writer to the Signet, was induced to give up business, and take a long lease of the estate of Durris, in Kincardineshire, upon which he expended great sums of money in improvements. But when the time approached for reaping the benefits of these, the lease was set aside, and the estate carried off by an heir of entail, leaving Mr Innes, senior, with a very slender equivalent for all the time and money he had thus expended.

One good thing resulted from this calamity. It brought out the native courage and vigour of Mr Cosmo Innes's character, and forced him to grapple manfully with his difficulties. His motto in such circumstances might well have been *Tu ne cede malis; sed contra audentior ito.* He never sat down with a listless look or a desponding heart, but turned to the first opening he could find that promised an escape from trouble. And here, as she generally does, Fortune favoured the brave, and gave our friend both a stimulus and an opportunity for exertion that might not otherwise have existed.

Another advantage that arose from the strong interest felt by all who saw his position, was that it excited the sympathy and attention of many friends of great influence and value. Much the most important of these, and one who greatly moulded and affected his future career, was Mr Thomas Thomson, whose acquaintance he formed in the year 1824, and with whose labours he became, for a long period, substantially identified.

Thomas Thomson was one of the most able and learned antiquaries and "Record Lawyers" that Scotland has produced, and he would probably have been recognised as the greatest among them, if his efficiency had not been marred or impaired by some defects of character and peculiarities of taste which interfered greatly with his practical powers. His fastidiousness, his aversion to hasty or ill considered opinions, and his general tendency to procrastination, led him to allow duties to stand over that should have been instantly and resolutely performed. As a member of the "Record Commission" he became busily occupied in the arrangement of the Ancient Records and Muniments of Scotland, and the publication of the old Acts of Parliament of the country came to rank as the "magnum opus" of his life. At the time when Mr Innes became acquainted with him, he was completing, or had completed, the eleventh volume of that collection, but the first volume of it had not been begun, being the portion of the work attended with the greatest difficulty, involved in the deepest obscurity, and for which new materials were daily coming to light from sources hitherto undiscovered.

The character of Mr Thomson, and his eventful history, full of varied incidents, some of a most pleasing, and some of a most painful kind, are exhibited in the interesting Memoir of him written after his death by Mr Innes, at the request of Mr James Craig. The latter years of Mr Thomson's life were obscured by no ordinary gloom of misfortune. In his administration as a "Record Commissioner," and as "Depute Clerk-Register," his accounts were allowed to run into great arrear and confusion, and attention came at last to be called to them by the officials connected with the financial departments of the Government. There had, undoubtedly, been great neglect, and considerable disregard of the proper limits of expenditure, which it was found wholly impossible to justify, but which, I am satisfied, would all have been put right by Mr Thomson and his many friends, if time had been allowed. But some of the officials concerned, particularly the men of mere routine, were too peremptory, and too punctilious, to look to anything but purely arithmetical considerations, and that, perhaps, took place which is not unfrequently observed, that injustice is done to a man by his political friends for fear of their being supposed to show him undue favour by protecting him from attack. However this may be, a step was taken which, in the opinion of many, was greatly to be deprecated.

A *criminal* charge was preferred against Mr Thomson for *defalcation* in his accounts, and it became necessary for him to appear for examination before the sheriff under that charge. At this time a change of government took place, and it happened that, as an official under the new crown authorities, I was entrusted with the duty of conducting Mr Thomson's examination. It was carried out with every degree of fulness and particularity, and I had much satisfaction in being able to report to my constituents that there were no grounds for a *criminal* charge. Mr Thomson had been guilty of laxity and carelessness, he had sometimes mistaken and exceeded his powers of expenditure,. and he had ventured upon disbursements for what he considered to be important objects not authorised by the strict letter of his instructions. But there was no trace of anything corrupt or fraudulent, and the application of the *criminal* law to his case appeared to mea harsh and inappropriate proceeding. These views were adopted by the crown counsel of the day, and Mr Thomson was liberated from any responsibility beyond the civil consequences of his pecuniary errors. It was impossible, however, that such occurrences, overtaking a man of Mr Thomson's high position, unblemished character, proud feelings, and eminent public services, should not be overwhelming, particularly at the advanced period of life which he had reached. The whole colour of his existence was thus changed; he had lost his office of "Clerk Register," and although he retained that of "Clerk of Session," the salary attached to it was appropriated to the discharge of his debts. "It was intimated to him at this time that another person was to be employed to complete the first volume of the Acts of Parliament." This is the language in which the occurrence is mentioned in the Memoir of his life. Mr Innes was the person so employed, and nothing could well be conceived more painfully interesting on both sides than the relation that came thus to exist between the pupil and his old master. Mr Thomson must have felt deeply the blow that thus deprived him of the opportunity of completing the crowning act of his long labours.

"He never again entered the Register House;" and Mr Innes adds, "that although he was generously communicative on every other point, where his assistance or advice was desired, he told me soon after I had been employed to complete the first volume of his great work, that *it must be a forbidden subject between us*."

In 1844 Mr Innes finished the first volume thus handed over to him, and did so in a manner which gained, I believe, universal approbation. I do not say that it was done as well as Mr Thomson at one time *could* have done it, but I am sure that it was done as well as Mr Thomson could *then* have done it, or rather, that the difference lay between its being done well by Mr Innes and its not being done at all.

The extinction that was thus given to Mr Thomson's efficiency in his peculiar department, for such was truly the result of these events, left Mr Innes as almost the only man in the field to whom either the public or individuals could resort for advice and assistance in matters of this kind, and he thus became one of our highest authorities on the subject of general or family antiquities.

It cannot be said, I think, that Mr Innes was ever successful as an advocate. He did not possess in a sufficient degree either what has been scornfully called the power "to make the worse appear the better reason," or which, I think, is its more correct description, the peculiar faculty on a properl deebateable question, to bring forward the fair and legitimate considerations that are to be weighed on either side. But he held successively important official appointments, that of Advocate-Depute, Sheriff, and principal Clerk of Session, the duties of which he discharged with adequate diligence. He was latterly appointed to the chair of Universal History in the University of Edinburgh, which was highly congenial to his general pursuits, and in which, I believe, he endeared himself to his students by his uniform accessibility and kindness, and by the valuable aid which he afforded them in their studies.

I have disclaimed any intention here of attempting to enumerate or estimate the different works of an historical or antiquarian kind which Mr Innes produced. I shall merely advert to his "Scotland in the Middle Ages," published in 1863, and his "Sketches of early

Scottish History," published in 1861, both of which are well known and are peculiar. Besides these, I may add in the words of Mr David Laing, which I am allowed to borrow, that "his labours in editing numerous volumes of ancient chartularies for the Bannatyne, Maitland, and Spalding Clubs, more especially those of Melrose, Moray, Holyrood, Dunfermline, Glasgow, and Kelso, as well as works connected with the public records of Scotland, will always be gratefully remembered." One of the works undertaken by him was the "Origines Parochiales of Scotland," which, if it could have been finished as it was begun, would have been a great and valuable work; but the difficulties in its execution proved to be far greater than had been calculated, and it remained at last in an unfinished state, which necessarily diminished its utility and importance.

I have always understood that the manner in which Mr Innes prepared the official works which he was able personally to accomplish, was much admired and approved of by the best judges both in this country and abroad, and in particular I have heard that M. Guizot, no mean critic, to whom he was personally known, always spoke highly of their merits. Partly on business exigencies, and partly as a form of relaxation, Mr Innes was latterly in the habit of visiting Paris in time of vacation, and greatly enjoyed the advantages of good Parisian society, as well as the opportunity thus afforded him of access to the French archives and other objects connected with mediæval history and antiquities. I may here observe that Mr Innes, among other accomplishments, had a very decided talent for letter writing, and that when he was abroad the accounts thus conveyed to confidential friends of what he had seen and felt on his travels, were a source of great interest and delight.

In Mr Innes's character—let me rather say within his bodily frame—two very different aspects of human power were to be seen. In the one we had a strong and athletic man, passionately fond of the country and country scenes, particularly those of this "Land of the Mountain and the Flood," the "Land of our Sires," excelling in all country sports, fishing, shooting, riding, coursing, and enjoying a pleasing though always a temperate repose from these exertions in some friendly or social meeting; while, in the other, we saw a man turned into a monk, busy among libraries and state records all day, and poring with double magnifiers and strong lamps till long after midnight, deciphering old and almost illegible manuscripts, and trying at once to master their character and make sense of their contents. These very different capacities and functions existed harmoniously together in the same individual, and instead of interfering with each other, communicated, perhaps, a mutual zest, and enabled the change to be pleasantly or at least contentedly acquiesced in. The versatility thus existing and kept up fitted him for a very varied and interesting range of social acquaintances, and of these he was always glad to avail himself in moderation. Nor was any one a more agreeable companion. His perfect good humour and good temper, his strong affection for his family and for his old friends, his never-failing courtesy, which arose from and indicated the chivalrous feeling that was at the foundation of his character, his utter absence of envy, jealousy, presumption, or self-conceit; and his sympathy with all innocent and gentlemanly relaxation and even merriment, endeared him to a very extensive and attached circle, and made his home the centre of much attraction and the scene of much social enjoyment. To these enjoyments his surviving friends still look back with unmixed pleasure and tender regard.

His literary productions, apart from those which appeared in an official form, show the same diversity of character to which we have already alluded. As specimens of these I may mention two excellent but very different papers, which a careless reader would scarcely conceive to have proceeded from the same mind: the one of these, a contribution to the "Quarterly Review" in 1843, upon the Ecclesiastical Antiquities of Scotland, and the other a paper inserted in the "North British Review" in 1864, on the Country Life of England. Each of these is well deserving of perusal, and the last mentioned is particularly interesting, as having first introduced into notice the achievements and writings of Charles St John, the well-known lover of sport, with whose tastes and habits those of Mr Innes were in full accordance, so far as circumstances would permit of their free indulgence.

Mr Innes's love for literature was strong and diversified. He was a fair Greek and Latin scholar. I hesitate to call him a *good* Greek scholar, as my old friend Archdeacon Williams denied that title to any one who did not know every good Greek author from Homer to Agathias. He was sufficiently at home in French and Italian to serve all the purposes which he had in view. But I think the books that he most loved were those that gratified best that chivalrous feeling that lay so deep in his heart. I remember as if it were yesterday hearing him read, fifty years ago, in an Italian society to which we belonged, the concluding character of Sir Lancelot, given in Malory's translation of the Morte d' Arthur, which runs in these striking terms:—" And now, I dare say, that, Sir Lancelot, there thou lyest; thou wert never matched of none earthly knight's hands. And thou wert the curteist knight that ever bare shielde. And thou wert the truest friende to thy lover that ever bestrode horse. And thou wert the truest lover of a sinful man that ever loved woman. And thou wert the kindest man that ever stroke with swerde. And thou wert the goodliest person that ever came amonge prece (press) of knights. And thou were the meekest man and the gentillest that ever eate in hal among ladies. And thou were the sternest knight to thy mortale foe that ever put spere in the rest!"

Mr Innes read these words with the greatest effect, but in that peculiar tone for which I think his reading was remarkable. He never read rhetorically, or in a declamatory style, but with rather a cold and dry manner, which, however, had the strange effect of leaving on his hearers a deep impression of his earnestness, and a thorough belief in what he said. It was impossible so to hear him without feeling convinced, as I then and ever was, that his own character involved in it many of those noble traits that the romancer described as forming the bright side of his hero.

Mr Innes's death was sudden, and took place at a distance from home, but it was calm and painless, and he was attended at the time by his wife and his only unmarried daughter. It is right to mention that in the later years of his life he enjoyed the advantage of a considerable accession of fortune, which came to Mrs Innes, and which placed them in comparative affluence. At the time he was taken away, his daughter was engaged under very happy auspices to the gentleman who has since become her husband, so that his departure took place amid circumstances that brought many consolations, and left little more in life to be desired.

3. Biographical Notice of Francis Deas. By the Hon. Lord Neaves.

Another loss to our Society which we have to record and to deplore at this time, arises by the death of Mr Francis Deas. This loss forms a striking contrast to that of either of the members of whom I have already spoken. They retired from the scene not prematurely, but full of years and well-deserved honours, having attained or approached the longer limit to which human life in normal circumstances is considered to extend; they had played out their parts, and, as having done so, were entitled to their dismissal amidst the plaudits of those who had witnessed and benefited by their labours. Mr Deas, on the other hand, was cut off, first by failing health, and ultimately by death, before he had attained the meridian of life, or could carry out into execution the capacities which, under a more favourable fate, would assuredly have earned him high distinction.

Francis Deas, the eldest son of the Hon. Lord Deas, was born at Edinburgh on the 1st July 1839. He went through the usual curriculum of the Edinburgh Academy, which he quitted in July 1856, having held a good place in all his classes, and having gained in 1855 the Ferguson medal, and in 1856 the Mitchell medal, both of them for proficiency in mathematics. He then went through the usual course of study at the Edinburgh University, taking prizes in almost all his classes—mathematics, logic and metaphysics, civil law, Scots law, rhetoric, and belles lettres, and natural philosophy; but he did not confine his studies to the usual routine. He was a zealous student with Professor Balfour for two or more sessions in botany, and accompanied him in his pedestrian excursions. He attended Dr Stevenson Macadam for practical chemistry, Professor Allman for natural history, and Dr Maclagan for medical jurisprudence. He continued in after life to keep up an intimacy with many of the Professors whose instructions he had thus received.

In 1859, before he was twenty, he went to Berlin, principally in order to perfect himself in speaking German, with which he was otherwise well acquainted, as well as with French and Italian. He attended law and other classes at Berlin University in summer 1859.

In 1860 he became acquainted with Sir David Brewster, having met him at his daughter-in-law Mrs Macpherson's house in Lasswade, and an intimacy and friendship sprung up between them, remarkable in several respects, and particularly in this, that young Deas was then barely one-and-twenty, while Sir David was in his eightieth year. The friendship thus formed subsisted during their joint lives, and was, I believe, a source of great pleasure and satisfaction to both, and certainly of great benefit to the younger of the two, though I venture to think that the benefit was mutual, as no one, and not certainly a man of Sir David's years and peculiar character, could fail to derive advantage from the simple and sincere affection of a youth so amiable and intelligent as Mr Deas. Sir David said of him from the first, that he had a more thorough and a more comprehensive hold of scientific principles than any man of his acquaintance not professionally scientific, and that he had so rare a combination of the faculties necessary for scientific research, that he (Sir David) deeply regretted "he was crippled by a profession so jealous as the law." Of the intimacy that thus arose very pleasing traces are to be found in the interesting volume of Sir David's Home life, by his daughter. In 1866 Sir David was seized when at Belleville with an unseasonable attack of hooping cough, and his illness was so severe as to excite the greatest alarm in Lady Brewster and his friends, although his mind remained bright, clear, and active. "A favourite young scientific friend," Mrs Gordon states, "Mr Francis Deas, was staying in the house at the time, and after hours of fatigue and suffering it was positive enjoyment to the invalid to make the little preparations for his visit, which was quite the event of the day. Believing himself a fast dying man, he left many instructions with Mr Deas as to the arrangement of his scientific instruments, &c., and two years afterwards, when the call really came, it was to this gentleman that he confided the finishing and reading of a paper for the Royal Society, which weakness prevented him from completing. It was on the Motion, Equilibrium, and Forms of Liquid Films."

Mrs Gordon gives us at the same time an interesting letter, written by Mr Deas to Mrs Macpherson after Sir David's death, sending his reminiscences of the three weeks spent by them together at Belleville on the occasion above referred to, and in a letter written by Sir David on his death-bed, he refers to Mr Deas as the friend to whom he had entrusted the final preparation of the paper on Films already mentioned.

I may add, that there was found in Mr Deas's repositories, after his death, a letter to him from Mrs Macpherson, Sir David's daughter in-law, giving an account of his last moments, and referring to the scientific subject in question, on which, I believe, Mr Deas read a paper in this Society as requested. That letter will be found in an appendix to the notice I am now reading.

Mr Deas was admitted a member of the Royal Society in 1867.

He had previously passed advocate in May 1862. At a later period, he was the first to receive the new degree of LL.B. (instituted in 1862.) Upon that occasion he was presented for graduation by Professor Lorimer, with a well-merited tribute to his diligence and proficiency in law. He had thoroughly studied his profession, and continued to do so, extending his attention at the same time to various kindred branches of study, such as medical jurisprudence and anatomy.

He began now to contemplate the publication of some legal work that should be useful to him, and prior to 1870 was engaged in preparing a second edition of Mr Fraser's work on "Master and Servant," which appeared in January 1872. His laborious application however to that task, carried on in conjunction with the practice which he was obtaining at the bar, seems to have injuriously affected his health, and to have made the first encroachment that appeared upon his constitution, and in the summer of 1870 premonitory symptoms were observed of that tenderness of chest which ultimately proved fatal. By advice of his medical attendants he went abroad, in order to make what is called the Nile journey. He had twice before been abroad, and was thus not an inexperienced traveller. He much enjoyed the voyage up the Nile to the Second Cataract, and took an interest in all that he saw, visiting all the objects of celebrity within his reach. The atmospheric varieties of the country, and in particular the pure and inspiriting air of the Desert, seem to have done him good, as well as to have afforded him pleasure. His journal consisting of memoranda during this voyage, of which I have seen a copy, is very interesting, particularly to those who knew him, and shows how his scientific tastes and feelings of curiosity were elicited at every step. It seems to be uncertain whether his health truly profited by this experiment. He appears to have doubted it himself; yet on arriving in London in June 1871, he wrote that there had been a marked change for the better ever since he recrossed the Alps, and that he was now so well that he wished to resume business. He returned to Edinburgh accordingly, and did resume business, but without attending the Parliament House. Upon putting out of hand his book on "Master and Servant," Mr Deas had commenced another work of a still more arduous kind, on the "Law of Railways," and to this he now applied himself as a professional task.

In February 1872 he again, by advice, went abroad, spending his time partly at Florence, but chiefly at Rome, still attending to all objects of interest, but at the same time continuing even there the progress of his book on Railways. He returned home in June of that year, perseveringly completed his book, and published it in January 1873. He very fittingly dedicated the work to his father, Lord Deas, "alike as a token of filial regard, and as a tribute to his acknowledged eminence as a lawyer." The book was received with great approbation, it evinces a wonderful degree of industry and energy, and cannot fail to be eminently useful to the profession, as many competent judges have gladly acknowledged.

In the narrative given above I have not said much of Mr Deas's scientific tendencies; but these, from the first, were very strong and decided. I have mentioned the opinion of Sir David Brewster as to the combination of qualities, which seemed peculiarly to fit him for scientific research, and his application to scientific subjects was constantly kept up. His reading was extensive in all the best books on science, and he contributed papers which were considered valuable to the best scientific periodicals of the day. He devoted a good deal of time to the study of optics, and had considerable practice in the use of the telescope; but was still more interested in microscopic investigations, in connection with which he amassed an extensive collection of objects for that instrument, nearly all prepared by himself, and accumulated during many years, wherever he travelled or happened to be.

It is to me a pleasant thing to record, and it must have been to his friends a great consolation to know, that in the midst of these scientific investigations, which were fearlessly and searchingly conducted, he never lost sight of those great principles that connect the works of the Deity with His personal existence and moral perfections. Many entries in his private memoranda show his fidelity to these feelings, and prove that he shared with his friend Brewster the reverence for a Supreme Power which that distinguished man always evinced in the prosecution of his varied inquiries. Mr Deas's reading on sacred subjects seems to have been much in the Book of Psalms, a book which has proved a treasure and a favourite study with all the devout admirers of nature; and he often expresses in his memoranda how much the admiration felt by the authors of that book for the works of the Creator would have been exalted and enhanced, instead of being deadened or destroyed, by the new wonders revealed through the aid of scientific instruments.

It was not only to professional and scientific subjects that he directed his attention. He had, I think, a genius for music, and performed on the pianoforte with perfect taste and with a degree of skill that was scarcely to be expected from an amateur who had so many other avocations and pursuits of a more urgent and engrossing nature. He was also fond of sculpture and painting, and his friend, Sir Noel Paton, seemed to have pleasure in sending him his paintings before they were despatched to London, at a time when Mr Deas was, from illness, unable to leave the house.

After what I have said, I think I may confidently claim your sympathy with me in this tribute to the memory of a young man for whom, when he was in life, I felt a strong esteem and regard, in whose sad fate I saw a great private and public loss, and whose memory, I think, is entitled to our affectionate remembrance. Looking to his natural talents and tastes, to the assiduous cultivation that he bestowed upon them, to the variety of subjects to which his studies extended, and to the high and sound principles with which his mind was imbued, I venture to say, that I know of *no* young man who, if he had lived and had preserved a sufficient measure of health, was more likely to extend the range and maintain the dignity of science, as well as of mental culture generally, while at the same time I cannot help adding, and there is a satisfaction even in this feeling, that I know of no one who, from the innocence of his character and from the purity of his feelings, as well as from the religious emotions which he carried into scientific investigations, was better prepared to be early removed from this temporary scene, seeing that such was the lot appointed for him. The loss of such a youth, who was doing so well, and promising so much more to be still done, must be a great affliction to all who knew him, and a very grievous one to those most nearly connected with him; but of such characters it is the privilege of survivors to speak, as good men have often done, that the memory of the departed is a treasure that cannot be outweighed even by present blessings.

APPENDIX.

Letter referred to on page 463, from Mrs Brewster Macpherson, found in Mr Deas's repositories, dated Allerley, Monday, February 10th, 1868.

"You will, I know, be intensely anxious to hear of dear, dear papa. Sir James Simpson says he cannot live over the night. We got a train straight on to Melrose on Saturday, so I gave my note to a porter to post for me. I hope you got it. We found Sir David much stronger and better than I had expected, so much so that I could not believe he was dying. He slept all that night, and up to twelve on Sunday. I could not believe he was dying, then he sank very rapidly. His perfect trust in the love of God, and in the finished righteousness of the Saviour, is wonderful. He has no wish to live—no fear of death—absolutely none. His faith is pure and childlike. His mind is perfectly clear. He expressed a wish twice to me that you should finish a paper which he had begun on Soap-bubbles, and read it for him at the Royal Society. He expressed the same wish to Sir James Simpson last night, and he has left a paper for you with instructions about it. Lady Brewster wrote at his request on Friday. He has spoken of you repeatedly to me with such kindness. Oh! Frank, it was awfully solemn all yesterday, and how much more so to-day—one of the great lights of the world going out."

4. Biographical Notice of Adam Black. By the Rev. Dr Lindsay Alexander.

Adam Black was a native of Edinburgh, where he was born on the 20th of February 1784. He received his education at the High School of the city, and afterwards attended for two sessions the classes at the University. Having selected bookselling as his profession, he became apprentice to Mr Fairbairn, an Edinburgh bookseller, and at the close of his apprenticeship spent two years in the house of Lackington, Allen, & Co., London. In 1815 he commenced business for himself in Edinburgh as a bookseller; and entered upon that career of wise and vigorous enterprise which he pursued to the end of his life, and in which, both as a man of business and as a public man, he earned for himself a wide-spread reputation. When the first Town Council under the New Municipal Act was elected, he was returned as one of the councillors; shortly after he became treasurer of the city funds, and laid the foundation of that scheme by means of which the pecuniary affairs of the city were at length brought into order, and the city relieved of the pressure of debt; and in 1843 he was raised to the office of Lord Provost, an office which he held by re-election for six years. On his retirement from this office he was offered a knighthood by the Government, but this he declined, alleging that as he was still in business as a retail bookseller and stationer, it would be incongruous for him to be standing behind his counter to be addressed there as "Sir Adam" by some boy sent up from the market "for a hard pen and a pennyworth of ink." In 1856 he was returned to Parliament as one of the members for the city, and to this dignified post he was repeatedly re-elected, and represented the city for nearly ten years. On his retirement from Parliament he still continued to take an interest in public affairs, as well as in the conduct of his business. For some years he had been withdrawing from bookselling and confining his energies and resources to publishing. By a happy union of boldness with prudence he raised his house to a foremost place among the great publishing firms of the country. Two large editions of "Encyclopædia Britannica," each of which was nearly all written anew, and numerous editions of the "Waverley Novels," and other writings of Sir Walter Scott, in various sizes and at prices that brought these matchless productions within the reach of all classes of the community, attest the vigour and skill with which he carried on his enterprise as a publisher. To him also is due the honour of being the first to summon the learning of the churches to the preparation of a "Cyclopædia of Biblical Literature," such as should present in a condensed form the results of the most advanced investigation into the history, literature, and archæology of the sacred writings. These are but a very few of the works he published, but they are the most important; of the rest it may be said generally, that they all possess some quality of excellence such as makes them valuable contributions to the literary or scientific products of the day.

Mr Black died on the 24th of January 1874, having nearly completed his 90th year. Not only for the services he rendered in various ways to the city, not only for his abilities and his success in business, not only for his enterprise and wisdom as a publisher, but still more for his moral qualities, his perfect integrity, his transparent honesty, his steadfast consistency, his unaffected piety, and his unswerving loyalty to truth and equity, will his name be handed down to posterity by the people of this city as that of one of the noblest and worthiest of her citizens.

5. Biographical Notice of Sheriff Cleghorn. By David Maclagan, Esq., C.A.

Thomas Cleghorn was born in Edinburgh 3d March 1818, and died there 13th June 1874. His father, Alexander Cleghorn, Collector of Customs, was an esteemed citizen of Edinburgh; his uncle, David Cleghorn, was long Crown Agent; and a second uncle, the Rev. Thomas Cleghorn, was parish minister of Smailholm, of which his great-grandfather, Dr Duncan, had also been pastor.

Mr Cleghorn was educated at the Edinburgh Academy and at the University, in both of which he was distinguished by earnest application and by high character. His favourite study was that of natural philosophy, and in the distinguished occupant of that chair, James David Forbes, he found a life-long friend and correspondent. Mr Cleghorn wrote a cordial and discriminating notice of Forbes after his death, in one of the magazines of the day. Like most of the foremost students of the University he was a member of the Speculative Society, and in later years, along with his friend Mr Robert Balfour, now deceased, wrote its history, a work of great research and interest.

Mr Cleghorn was called to the Scottish bar in 1839, and held successively the offices of Advocate-Depute, Registrar of Friendly Societies, and Sheriff of Argyle, which latest appointment he continued to hold until his death. He was unanimously elected in 1871 Legal Adviser of the Free Church of Scotland, of which he was an attached member and office-bearer. Mr Cleghorn's connection by marriage with the family of the late Lord Cockburn introduced him to a highly cultivated literary circle, in which he was well fitted, by his classical and scientific knowledge and wide range of literary study, to occupy a place. For very many years Mr Cleghorn devoted much time to the advancement of educational, benevolent, and religious objects, to all of which he was a most liberal contributor. The welfare of schools and colleges generally was always a source of interest to him, while the Edinburgh Academy, of which he was for many years a Director, and the University of his native city, were specially dear to him.

Wellington School, an institution for the reformation of young criminals, was founded by him, and to its support he largely contributed both means and personal labour.

Mr Cleghorn has left a name greatly esteemed, and will be remembered as a man of much culture and many acquirements, as well as a citizen of proved worth and of large hearted public spirit.

6. Biographical Notice of Henry Stephens. By Professor Maclagan.

Mr Henry Stephens was in the Royal Society essentially the representative of the important science of agriculture, and has left behind him a reputation as an agriculturist not confined to Britain, for his works on agriculture have been translated into every European tongue, and are thoroughly appreciated abroad. He was born in July 1796, in Forfarshire, where he inherited the estate of Balmadies. He seems, from his earliest youth, to have had an enthusiastic love for agriculture, and to have from the first regarded it not as a business to be conducted by empirical or routine rules, but as an art to be practised under the guidance of scientific principles. He intended that he should be a practical farmer, but he resolved that to fit himself for this he should make himself a well-educated gentleman. His motto seems all along to have been "thorough," and his guiding rules diligence and method. Nothing can illustrate this better than a manuscript volume which he left behind him, bearing on its title page, "A Course of Education, comprising Mathematics, Natural Philosophy, Natural History, Chemistry, and Agriculture. Dundee, 1815." The volume, which looks almost as if he intended it to be printed as a text-book for young agriculturists, was begun by him when he was 19. It is not original work, but consists of notes taken by him during his attendance on courses of instruction, of which he gives the following account in a formal preface to his manuscript volume:—

"The notes on mathematics, natural philosophy, and the outlines of chemistry, were taken at the lectures of Mr Duncan in the Dundee Academy, from 1st October 1809 to 1st August 1810, and from 1st October 1810 to 1st August 1811, which completed the session at the academy.

"The notes on chemistry were taken when attending the lectures of Dr Charles Hope in the University of Edinburgh, from 6th November 1812 to the 26th April 1813. Those on natural history, when attending the class of Mr Robert Jameson, in the same place and during the same period. In the same place the lectures on agriculture by Dr Andrew Coventry, commenced 5th January 1813 to 28th April of the same year; but during that period [I] attended his class twice a day, at 8 o'clock in the morning and at 3 o'clock in the afternoon."

This preface is a true index of the character of the man, even as he was known in his old age—complete methodicity, unsparing energy, and perfect precision in everything.

Stephens had, by theoretical preparation, made ready for cultivating his own estate, but he felt the necessity for practical study also, and therefore he placed himself, with a view to learning his work practically, with one of the largest and most skilful agriculturists in the county of Berwick, which had then the repute of being the best farmed district of Scotland. On this farm—Whitsome Hill—he remained for three years, engaging, as he himself records, "in every sphere of work which the ploughman, the shepherd, and the field-worker must perform in the field, or the steward or cattleman at the steading;" even in the dairy and poultry-house part of his time was spent; and all this he undertook "not of necessity, but voluntarily, and with cheerfulness, in the determination of acquiring a thorough practical knowledge of his profession."

Thus armed, he was prepared to encounter the work of cultivating a part of his own estate, and he soon saw that to do this satisfactorily a considerable expenditure of money was called for; and this was done, to the effect of raising the value of the farm which he personally worked, from L.150 to L.400 a year. But evil days were in store for him. By the failure of an Indian house in which his money was invested, and just at the time when he had spent much on improving his property, he was straitened in his means, and he had to bethink himself of other ways of carrying out his life's object of being an agriculturist. It was at this time, when he was under the cloud of misfortune, that an accident occurred which laid the foundation for his reputation as an agricultural author. He was travelling in the coach from Dundee to Edinburgh when he encountered, as travelling companion, the eminent founder of the great publishing-house of William Blackwood and Sons. The sagacious William Blackwood was too acute not to perceive that in his young travelling companion he had found a man thoroughly versed in the science of agriculture. He shortly after called Stephens to his aid in conducting the Journal of Agriculture, and thereby was commenced a literary connection with the Blackwoods, which has extended even to a third generation. It was through them that he gave to the agricultural world his "Book of the Farm," the first edition of which was published in 1842, and a second edition in 1871—the manuscript of which,—almost a complete re-writing of the original edition,—was worked up with the same precision, attention to detail, and neatness of penmanship, which characterised the "Course of Education" of 1815. His other works were—in conjunction with Mr G. H.

Slight, "The Book of Farm Implements and Machines;" in conjunction with Mr R. Scott Brown, the "Book of Farm Buildings;" in conjunction with Dr Seller, "Physiology at the Farm;" the "Manual of Practical Draining;" the "Yester Deep Land Culture;" and the "Catechism of Practical Agriculture."

He was an original and active member of the Meteorological Society of Scotland, and, although not writing much on the subject, he was in constant communication with the Secretary of the Society, especially in giving advice and assistance in all questions of meteorological science which had a special bearing on agriculture.

Mr Stephens, for many years previous to his death, was in the habit of repairing annually for the recruitment of his health to Homburg, and, in the course of his various visits to Germany, visited all the more celebrated vine growing districts on the Rhine. He carried his agricultural spirit with him in all these trips, noting all the processes of vine cultivation, even to its minutest details, and bringing back with him an ever increasing appreciation of all the best vintages of the Rhine, of which he always possessed a modest but select store, with which he delighted to refresh any friend visiting him at Redbraes, whom he thought capable of fully estimating his favourite wines. He had, however, even better entertainment for his visitors in his conversation, which was to the last full of good nature, with a large spice of "pawky" humour, sometimes in his later years a little prolix, but always yielding something in the way of anecdote or scientific—especially agricultural—observation worth listening to. For many years he had been made aware that he had a certain amount of organic change of structure in the aortic orifice of the heart; but this made no progress, and, so far as it was concerned, he might have prolonged his days. His death, however, was ultimately due to accident. It is remarkable that he was three times the subject of poisoning. He was one of the first of several instances which have occurred of poisoning by the flesh of American partridges, and his case was graphically narrated by his then medical attendant and friend, the late Dr Burt. He, on another occasion, suffered a good deal by the inhalation of coal gas which had escaped in his bedroom during the night, but from this he soon got well. It was, however, a repetition of this accident which ultimately led to his death. On the night or early morning of 21st June 1874 he had, as he thought, extinguished the gas in his small bachelor bedroom, but unfortunately had left the stop-cock open, and it was his not making any movement in the morning that attracted the notice of the servants; one of them entering his room found him insensible, in an atmosphere strongly charged with gas, and, seeing at once what had happened, sagaciously opened the window, and got him to swallow some stimulant. His medical attendants succeeded in rousing him from his comatose state, and he seemed in the fair way of recovery, but a low congestive inflammation of the lungs supervened, and proved fatal on the 4th of July.

7. Biographical Notice of Christopher Hansteen. By Alexander Buchan, Esq.

CHRISTOPHER HANSTEEN was born at Christiania on the 26th of September 1784. In 1802 he entered the University of Copenhagen as a student of law, which, however, he soon abandoned for what was to him the more congenial study of mathematics. He became mathematical tutor in the Gymnasium of Fredericksburg, in the Island of Zealand, in 1806, and about the same time he gained the prize which had been offered by the Royal Society of Sciences of Copenhagen for the best essay on terrestrial magnetism. Shortly thereafter, viz., in 1814, he was appointed to the chair of astronomy in the University of Christiania, which had recently been founded by Frederick VI. of Norway.

He continued to prosecute his researches into terrestrial magnetism with ardour and success, the results of which appeared in his great work, entitled " Untersuchungen über den Magnetismus der Erde," which was published in 1819 by the liberality of the King of Norway. The work was illustrated with an atlas of maps, and besides containing the fullest and best collection of observations on terrestrial magnetism which had then appeared, it was remarkable for great breadth of treatment and sound philosophical generalisations.

In continuing the prosecution of his physical researches, he made a journey into Siberia, accompanied by Ermann and Due, the expenses of the expedition being defrayed by the Norwegian

Government. One of the most important results of this expedition was the establishment, on Humboldt's recommendation, of the ten magnetical and meteorological observatories by the Emperor of Russia, at which hourly observations were recorded for many years, and annually published *in extenso* by the Russian Government, the whole forming the completest record of these phenomena we yet possess.

Shortly after his return from Siberia the Norwegian Government voted the funds for building an astronomical and meteorological observatory at Christiania, which was erected under Hansteen's direction. He also superintended the trigonometrical and topographical survey of Norway, which was begun in 1837.

The completion of his fifty years' public services was commemorated in 1856, shortly after which he ceased to lecture, and in 1861 retired altogether from public duty. He died on the 11th April 1873, at the advanced age of 88.

8. Biographical Notice of Jacques-Adolphe-Lambert Quetelet. By Alexander Buchan. Esq.

JACQUES-ADOLPHE-LAMBERT QUETELET.—On 17th February 1874 Quetelet died at Brussels, in the seventy-eighth year of his age, having been born at Ghent on 22d February 1796. At the age of 18 he was appointed Professor of Mathematics in the College at Ghent; and in July 1819, the degree of Doctor of Science was conferred on him by the University of the same town, which had just been founded by King William. His dissertation on the occasion was so well received that he was shortly thereafter appointed to the Chair of Mathematics in the Royal Athenæum of Brussels. In February following he was elected a member of the Academy of Sciences and Belles-Lettres.

The earliest of Quetelet's published memoirs, which began to be issued in 1820, were on geometrical subjects. He soon, however, directed his attention more exclusively to physics and astronomy, and lectured publicly on these subjects with great success.

In 1823 he was sent to Paris to report on the observatory of that city, for the guidance of the Belgian Government in founding a similar observatory at Brussels. After some delay the observatory was founded, with Quetelet as director; and in 1833 were begun the valuable series of astronomical, meteorological, and other physical observations for which this observatory is so favourably known. Of the work done by this observatory, special mention may be made of the catalogue, begun in 1857, of stars which seem to have appreciable motion; and the systematic observation and publication, from 1836, of the occurrence of meteors and shooting-stars,—records which proved to be of so great value thirty years later when the true character of these bodies was satisfactorily established. The meteorological observations have been particularly full and valuable, and they have been exhaustively discussed by Quetelet in his "La météorologie de la Belgique comparée à celle du globe," published in 1867,—a treatise which must yet be regarded as the fullest and best account of the meteorology of any single locality on the globe. Stations at Liège, Ghent, and other places in Belgium, were also established by him in 1835.

He was elected Perpetual Secretary of the Academy of Sciences and Belles-Lettres in 1834, and to his influence was chiefly due the section on the Fine Arts which was added to the Academy in 1845. To this section Quetelet made extensive and original contributions, particularly in his researches regarding the proportions of the human body, the results of which are published in his "Athropométrie." In matters referring to the higher education of the people, the census, and several other national questions, the Belgian Government availed itself repeatedly of his great knowledge and experience.

He was made President of the Central Commission of Statistics at its establishment in 1841, and continued President till his death. His first paper on Statistics was published in 1826; in 1835 appeared his "Physique Sociale," and ten years later his "Lettres sur la théorie des Probabilités appliquées aux sciences, morales, et politiques." He originated the idea of convening an International Congress of Statistics, and the first Congress was held at Brussels in 1853.

The many-sidedness and fertility of Quetelet's genius may be seen from the list of his scientific memoirs, enumerated in the Royal Society's Catalogue, amounting at the close of 1863 to 220. It is in the field of statistics that Quetelet appears as a great discoverer, and his success in this department is to be attributed to the clearness with which he saw that statistics occupy the same place in the development of the social and political sciences that observational data do in the development of such sciences as astronomy and meteorology; to the patient industry with which, through long years, he gathered together his facts; and to the mathematical skill he brought to bear on their discussion. He was truly, as stated by the Academy of Berlin in their congratulatory letter on the occasion of the centenary of the Belgium Academy, "the founder of a new science, which proceeds from the firm basis of observation and calculation, to discover and unfold those immutable laws which govern the phenomena, apparently the most accidental, of the life of man, down even to his most trivial actions."

9. Biographical Notice of George Berry.

By George Barclay, Esq.

Mr **George Berry** was born in Edinburgh (where his father, of a Quaker family in Somersetshire, had settled as a merchant), on the 12th of January 1795. Bred to business himself, partly at home and partly in France, Mr Berry succeeded his father in Edinburgh, but about 1834 removed to Leith, whence, after a successful mercantile career of twenty years, he retired, and died at Portobello on the 1st of May last.

While in Leith Mr Berry took an active part in public affairs; he was one of the founders of the Chamber of Commerce, and having early become an enthusiastic "Free trader," he continued, during the years of struggle which preceded the national adoption of that policy, perhaps the most prominent representative of free trade doctrines in Leith.

But though greatly occupied with business, Mr Berry was through all his life also somewhat of a student. A great reader, and gifted with a retentive memory, he was well versed in English literature and in science. He had been a pretty good chemist of his own day, but specially a devoted and accomplished mineralogist and geologist of the school of Jameson. In pursuit of these studies he spent for years as a young man his spare hours at home, and his holidays in wanderings after "specimens," in the then little travelled Western Highlands, of which he had many curious anecdotes to tell; following his master, he became a keen "Wernerian" in those days of hot geological controversy. He was admitted to the membership of the Royal Society in 1861.